technological advances by moving rapidly from concept into our daily lives.

Technology moves in cycles, as do most important things, and the down cycle is the proper time to prepare for the next period of discovery and development. Things have been slow in the exciting development business recently. This slack time is just what we need to give us a running start on the hectic days that we can be certain are waiting for us in the future. Let it not be said that the society which dawdled for nearly 200 years over the application of electricity was guilty of a similar lapse in the use of superconductivity; and extended space travel, colonization, and manufacture. As did electricity, these technologies hold the prospect for tremendous change in our lives and economies, now and for centuries into the future.

Tomorrow belongs to those who prepare today. And the nature of tomorrow, even its time of arrival, are entirely in our hands. We can wait patiently for tomorrow to happen eventually and react to it when it does arrive, or we can prepare now to make it happen soon, and in *our* way.

Granted, we don't know all that there is to know about superconductivity or high-power lasers or extended space travel or controlled fusion or the potential impact of advanced computer capabilities, but we know enough that plans can be made. The implications for society and technology can be surmised. The needs as well as the opportunities can be foretold. University programs can be started now to produce expertise in these critical areas of social and scientific planning. Industry will want to have the capability and experience to implement these next great strides.

The theories have been derived. Technology is advancing in all of these areas.

Opportunity approaches our door.

Will we be ready for tomorrow?

Breakthrough in Understanding Plasma Structures

Exclusive to NSIPS

In the past five years, an increasing number of scientists have come to recognize that the new, unexplored frontier of science lies in the field of plasma physics. Because of the urgency of realizing controlled nuclear fusion, a technology based on plasmas, the scientific challenge of these frontiers assumes an even greater importance. In the last nine months there have been a series of theoretical advances in forefronts of research into "non-linear phenomena" in plasmas; one of these, which was published by G. Lamb, Jr. in Physical Review Letters last August, may be the first step in a profoundly significant attack on the problems of understanding plasmas.

The characteristic feature of non-linearity, in whatever field it appears, is "progress." In social systems, history is made by the steady development of new technologies, new concepts, and their applications. In the geological sciences, the "evolution" of the Earth has been a change from a uniformly hot, smooth planet, to the present world of highly differentiated atmosphere, climates, and elevations. A similar development is obvious in the biological sciences in the first origins of life from inorganic matter, and the subsequent coming into being of more and more complex living things, which make up a biosphere supporting greater and greater energy flows. For the first time in the physical sciences, plasma physics now seems to have demonstrated similar phenomena occurring in a collection of non-living matter.

A plasma is a gas which is so hot that the molecules in the gas have been split into their constituent atoms, and these atoms have then stripped of their electrons. The resulting matter is ionized and interacts with itself by means of electrical and magnetic forces, which are thousands of times stronger than the molecular forces that determine the properties of a usual gas. The key quantity in determining the astounding properties of a plasma is its "energy density," the amount of energy per unit volume that the plasma contains. The electromagnetic interactions and the high energy density of a plasma result in two quite extraordinary qualities:

1. The plasma has a tendency to evolve from states without order to states which have large-scale organized structures. An initially randomly forced plasma can, for example, spontaneously transform itself into a collection of large whirlpool motions. The spontaneous formation of vortices is a persistent feature of the natural evolution of a plasma.

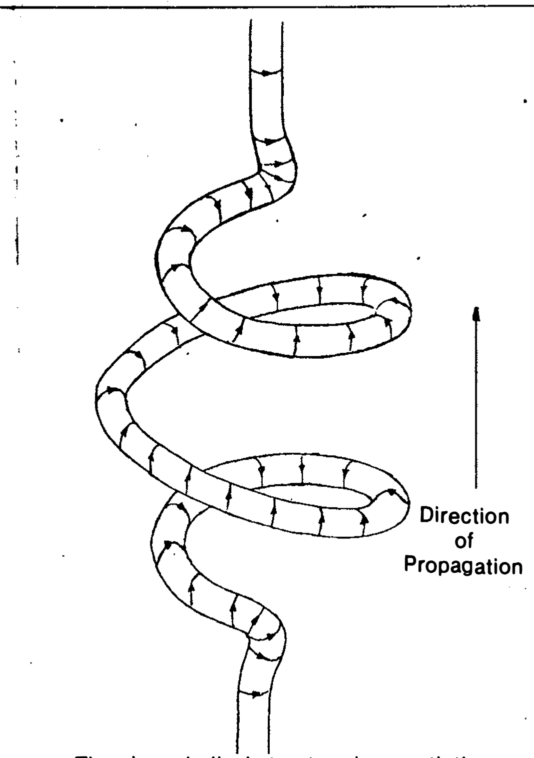
2. This tendency toward the formation of large organized structures, like vortices, out of random motion in a plasma is a specific example of the striking characteristic of a plasma to concentrate its own energy. In the usual situation of a low-temperature gas, or, indeed, any low energy-density material, the energy available to it is distributed in a uniform way throughout its volume. But plasma, on the contrary, will bunch up the energy to the point that the energy density in one of the structures can be 100,000 times as great as the initial state — the ideal conditions for producing controlled thermonuclear fusion reactions.

A general theoretical concept of this sort of behavior does not exist at present, but the name given to this whole class of "anomalous" behavior is "non-linearity." This concept, while clearly important, is still in a formative stage. At this point it appears that the key to understanding non-linearity lies in understanding the tendency that highly energetic plasmas have for forming self-ordered structures. What seems to be required from an experimental and theoretical research program is a kind of zoology of these structures. With this zoology, the truly fundamental problems of mathematical physics can begin to be studied — the zoology is the first experimental step.

Solitons, Cavitons, and Other Structures

There are two types of structures that appear in plasma dynamics, and these two classes of structure have seemed up until now to be entirely different. The first of these are called hydrodynamic, or MHD (for magnetohydrodynamic), because of their similarity to structures that arise in fluids, and include vortices, filaments, rings much like smoke rings, and galaxy-like structures. The second general class arises out of plasmas created by electron beams or lasers, which produce spikes of electrical field ("solitons"), small evacuated cavities ("cavitons"), and self-accelerating bunches of particles.

The physical mechanisms between these two kinds of phenomena are entirely diferent. The MHD structures come out of the motion of the heavy component of the plasma, the ions, and, seem in fact to depend on electrons in an entirely unimportant way. The wave motion that is associated with these structures (ion acoustic waves and Alfven waves) have a slow time variation, and the time scale of the phenomena tends to be relatively long. In the case of the solutions, however, the ions can be thought of as stationary, and the important dynamics come from the much more rapid motion that the electrons, being much lighter, undergo; the waves and other collective modes of the plasma in this regime (Langmuir waves) are rapidly varying. In spite of this deep physical difference, both kinds of plasma dynamics, the MHD and electron regimes, give rise to self-organized structures in a hot plasma. This fact has been a persistent difficulty in understanding the significance of solitons, vortices, and their relatives: what could be the fundamental connection between phenomena which are qualitatively similar (in that they are both examples of self-generated structure that a plasma creates), but which have totally dissimilar physical origins? What kind of property of the plasma are they evidence of? Or, perhaps, is it only accidental that both occur, and is it a kind of anthropomorphism that they would be thought similar at all?



The above helical structure is an artist's rendition of soliton propagating on vortex filament.

Lamb's work makes an important first step in answering these questions, for he has shown that there is a systematic connection between vortices and solitons, at least in fluids, and, more importantly, has identified the underlying feature of the fluid which creates this connection between solitons and vortices — the intrinsic geometry of the fluid motions.

Solving the NLSE

In the following description of Lamb's work, I will try to make the description understandable without reference to the mathematical formalism that Lamb uses, but the equations that are central are reproduced so that those with mathematical background can duplicate Lamb's formal reasoning. The key equation in the study of solitons in a plasma (although it is not by any means the only equation which gives rise to solitons) is the non-linear particle differential equation called the non-linear Schrödinger equation (NLSE):

$$i\frac{\partial}{\partial t}\psi + \frac{\partial^2}{\partial x^2}\psi + 2|\psi|^2\psi = 0$$

This equation describes the behavior of the amplitude, or strength, of a wave, and shows that the time variation of the wave (given by the time derivative) is dependent on a "kinetic energy," given by the spatial derivative, and a "potential energy," due to an attractive force which increases as the strngth of the wave increases. This is the term in the equation which depends on the third power of the amplitude. Physically, it says that the wave is attracted to places where the wave is already strong — and hence, the wave has a tendency to bunch up and concentrate itself into sharp spikes of wave energy which are called solitons. The NLSE has another set of solitons which are called plane waves, and correspond to what we usually think of as light rays. However, in a plasma, light waves (and other waves) travel according to the NLSE, so that if they enter the plasma as a plane wave (or usual light ray), they tend to break up into small filaments of concentrated light, solitons, as the NLSE's "potential energy" forces the light to bunch up. In the laboratory this class of solitons occur persistently whenever a strong laser is used to illuminate a plasma — hot spots of self-focused laser light are created.

The NLSE is a very difficult equation to solve and its general solution continues to evade scientists and mathematicians. However, in 1972 two Soviet physicists, V. Zakharov and A. Shabat, discovered a powerful tool for dealing with the NLSE. The method, called the "inverse scattering method," allows the NLSE to be reduced to two *linear* partial differential equations, which are then much easier to solve. The problem with the inverse scattering method, however, is that the transformation that reduces the NLSE was found by intuition, and attempts to apply the method to other non-linear partial differential equations have depended on guessing an appropriate transformation — there is no systematic way of connecting the transformation with the equation. Lamb's contribution is specifically to show a physical connection between the NLSE and the transformation required for the inverse scattering method — and he invokes the physics of vortices to make this connection!

Lamb's argument requires three steps:

1. Start with the physical situation of a long, thin whirlpool in a fluid, a "vortex filament."

2. Derive the equations that describe the motion of the vortex filament in the "intrinsic" geometry of the filament. This means using a measuring system for describing the filament that is defined in terms of the curvature and "twistedness" of the thread of vorticity.

3. Solve this equation. When this is done, it turns out that the motions that can occur on a vortex filament are described by the NLSE and that a plane wave corresponds to a smoke ring (where the two ends of the filament have been joined together) and a soliton corresponds to a loop of helical motion (see figure). Once the NLSE is applied to this situation, there are simple geometrical constraints that apply in an obvious way to the filament, which give rise to the inverse scattering method. Put differently, the intrinsic geometry of a vortex shows how solitons behave.

The idea of a vortex filament is due to Helmholtz, who published a paper in 1858 in which almost all that was known until recently about these filaments wsa derived. He showed very generally that all vortex motion in a fluid was expressible in terms of vortex filaments, and derived very general results about their motion. The central idea in Lamb's work is to imagine the vortex filament as a "space curve," that is, a line moving through space, which can turn and twist as it moves. The so-called intrinsic geometry of this space curve is then given by measuring the curvature (how sharply the filament turns at any point) and its torsion (how twisted it is at any point).

Geometries

These two quantities are expressed in terms of a coordinate system that moves with the twisting and turning filament. The effect is just the *opposite* of a gyroscope on a ship. The gyroscope gives the navigator on a ship, now matter how the ship sways, a constant bearing; the intrinsic geometry, on the other hand, defines the position of the ship in terms of all the twists and turns that it has made. All the geometric qualities that are required can be expressed in this geometry that travels with the filament, and R. Betchov, in 1965, was able to write down the set of complicated equations that express the motion of the vortex filament in this intrinsic geometry. At this point it is not clear why the intrinsic geometry would be informative. It is certainly more complicated than an absolute geometry (think of a ship trying to navigate by keeping track of all the twists and turns rather than using the North Star), but — and this is the key to Lamb's work — the simplest *physical* reasoning comes from realizing that the geometry that a physical object moves in is as much a product of the dynamics of that object as its motion; geometry is a dynamic entity. In other words, the physical laws that prescribe the behavior of an object at the same time create a space in which that motion takes place.

Betchov's equations, which combine the physics of the vortex filament in the intrinsic geometry of the filament (conceived as a space curve), are expressed in the three vectors that travel with the filament and define the height, width, and depth at that point on the filament. Once these equations are known, it is then possible to derive the equations for the curvature and torsion of the filament. Lamb's work depends on an extraordinary fact: if Betchov's equations are used to write the equations for the torsion and curvature, the result is the NLSE! Specifically, if we define:

$$\psi = \frac{\kappa}{2} e^{i\varphi}, \quad \frac{\partial}{\partial x}\varphi = \tau$$

where:

κ = curvature of the filament,
τ = torsion of the filament,

then:

$$i\frac{\partial}{\partial t}\psi + \frac{\partial^2}{\partial x^2}\psi + 2|\psi|^2\psi = 0$$

This is the NLSE written for the above combination of the torsion and curvature of the vortex filament.

This is a very powerful result, since we have a specific model of how the NLSE equation comes out of the intrinsic geometry of the vortex filament, and the geometrical information can be used to study the NLSE. Lamb did this, and noticed the very simple geometrical fact that the three vectors that define the intrinsic geometry, since they are perpendicular and have a specified length, put constraints on the possible values of the torsion and curvature. This, in turn, means that the information from these constraints puts restrictions on the NLSE. Specifically, it allows us to define a transformation that takes advantage of the restrictions to reduce the NLSE to a set of linear equations. When this is done, the resulting linear equations are those of the inverse scattering method. The result not from an inspired guess, however, but rather are a natural and, in this case, simple result of the geometric considerations. (Here is a good example of the power of intrinsic geometry. Its use laid bare the inner workings of the filament).

Where does this all lead? Lamb has shown that two things are of importance: First, the phenomena of solitons and vortices are intimately connected. In a fluid, the dynamics of vortex motion generate the same equation that describes solitons in a plasma. Second, the connection between solitons and vortices comes out of the intrinsic geometry of the vortex filament. It is a result which depends on using a geometric kind of physics. The actual whirlpool geometry of the vortex defines the mathematical geometry natural to a soliton.

How does this relate to phenomena in a plasma? This insight opens up the possibility of a systematic study of the relationships that exist among phenomena like solitons and vortices, in whatever medium they occur. But it is only a first step. The dynamical equations, and hence the geometry, of a plasma are much more complicated than that of a fluid, and so the problem of extending Lamb's result in a rigorous way to a plasma is a difficult one. Lamb has begun the kind of comparative anatomy of the organized structures generated by a plasma. Continuing this research, both experimentally and theoretically, is the key to one of the most exciting frontiers in science.