A topological shock-wave model of the generation of elementary particles

by Jonathan Tennenbaum

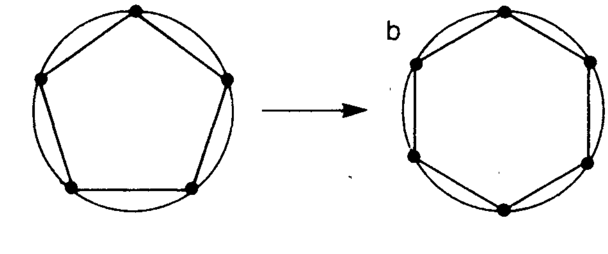
Dr. Tennenbaum is a mathematican currently editor of the European, German-language edition of the scientific journal Fusion. *The following paper was submitted on Dec. 22, originally intended to serve as a preliminary report on the feasibility of part of a research-project proposed by LaRouche-Riemann model international task-force leader Lyndon H. LaRouche Jr. Although the paper is technical, the subject is of such importance and practical relevance to various areas of ongoing research, we have chosen to publish this preliminary paper here as an aid to accelerating participation in this important exploration.*

My adopted task is to synthesize a simple geometrical model, combining the notions of shockwave and harmonic ordering of space.[1] This paper outlines the initial choice of approach being taken at the present time.

The harmonic ordering of space effectively translates into terms of quantities of the modes of appearance of the universe. Illustrative of this is Kepler's classical argument, that the possible orbits of planets are limited to a discrete set of possibilities. The shock-wave must then correspond to the "quantum of action" by means of which the universe moves from one harmonic ordering to another.

The simplest form of harmonic ordering is that defined by the division of the circle's circumference by means of inscribed regular polygons. In those terms of reference, for example, how could a shock-wave be seen as characteristic of the process of transformation of a pentagon into a hexagon **(Figure 1)**.

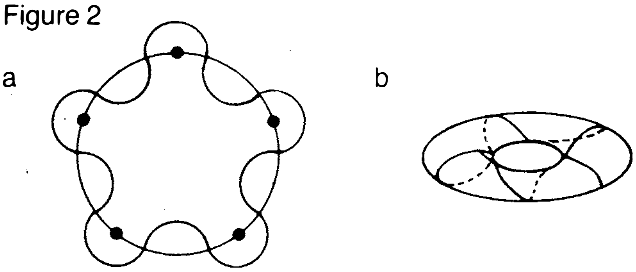
Figure 1



To deal with this initial choice of statement of the problem, I first consider the divisions of the circumference of the circle as a problem in topology. Namely, I think of these divisions of the circumference as being generated by a "wave" on that circumference.

This is equivalent to considering closed curves on a torus **(Figure 2)**. From the standpoint of topology, the closed curves

Figure 2



on a torus are quantized by their winding-number: the number of times they wind around the torus in a single, completed cycle. That number, then, corresponds to the number of divisions of the circumference of the corresponding circle, to the number of completed cycles of the wave in each completed orbit. Any two curves with the same winding-number are topologically equivalent, in the sense that each can be continuously deformed into the other.

From this standpoint of reference, look again at the problem implied by proposing to transform a pentagon into a hexagon. How might we conceive of a shock-wave which would accomplish the *discontinuous* transformation required to change a given curve into one of a higher (or, lower) winding-number?

For illustration, consider a simple case. Consider the equivalent case, of a flexible curve on the exterior of a cylinder **(Figure 3a)**. Imagine this flexible curve to be a rubber band first placed along the surface along a line parallel to the cylinder's central axis. Then, think of stretching this rubber band, to convey the idea of a deformable curve. In the first approximation **(Figure 3a)**, this deformable curve has a

Figure 3

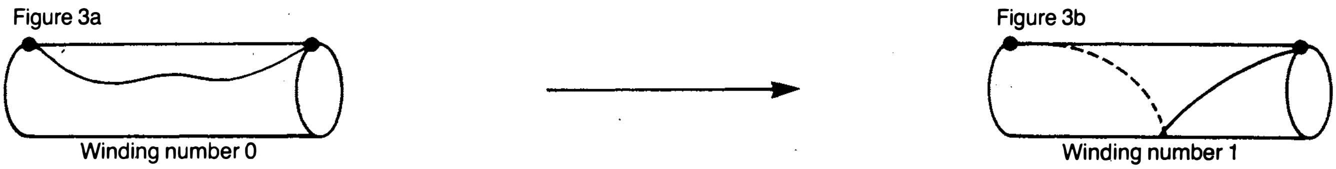
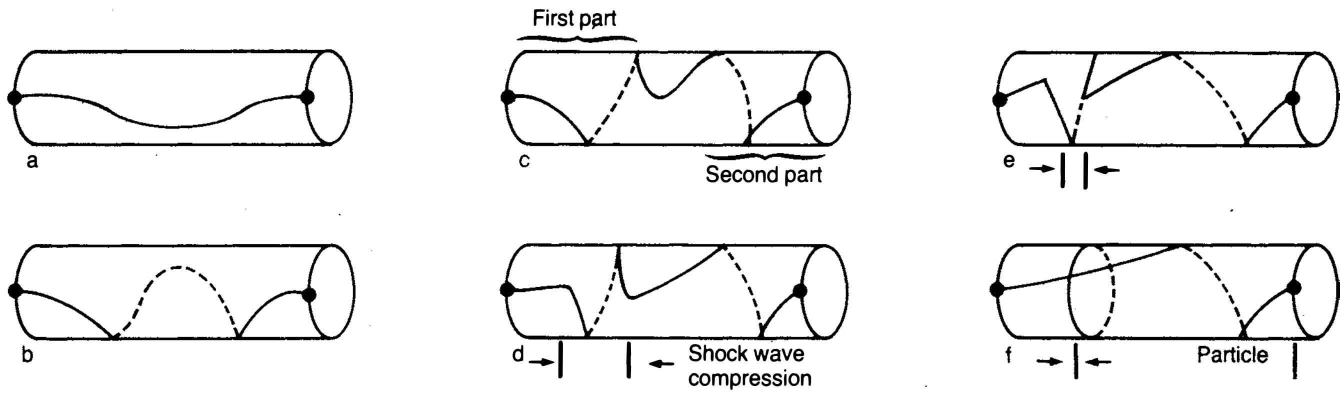


Figure 4



winding-number of the value "0." The problem implied, of transforming a pentagonal into a hexagonal division of the circumference of the circle, is therefore to be seen as of the form of transforming **Figure 3a** into **Figure 3b,** whose curve has a winding number of value "1."

In the simplest illustrative case, we require a "shock-wave transformation" which transforms the "zero division"—the case of winding number = 0—into a curve which encircles the cylinder a single time.

I propose the following process: we deform the curve into a path which first goes around once in a clockwise direction, and then doubles back, to go around once in the opposite direction. The winding number remains zero. Then, we imagine a *shock-wave-like process,* by which the first part of the curve is "compressed," until, with formation of the shock-front, this part is "fused" into a closed circle intersecting the remainder of the curve (**Figures 4-a, 4-b, 4-c, 4-d, 4-e, and 4-f**).

As a result (**Figure 4-f**), we have a particle, represented by the circle, which is a determined, localized singularity on the axis of the cylinder, and we have also a curve which encircles the cylinder once in the counterclockwise direction. We could think of this latter as the field associated with this determination of the particle—as, in that sense, "generated" by the particle. The projection of this complex-number wave process onto a plane parallel to the axis of the cylinder can now be described (**Figures 5-a** through **5-f**). I indentify the intersecting line in **Figure 5-f** as corresponding to Leibniz's "delta."

The general repetition of this process yields an arbitrary winding-number, accompanied by a number of particles corresponding to that winding-number.

If a particle is "untied," the winding-number drops back by 1. A similar process can be used to decrease the winding-number of a curve, by creating "anti-particles" as compressed loops running in the opposite direction. Or, we may begin with any winding-number, and create a "pair" (mathematically analogous to the electron-position pair-production actually observed) of opposite-signed singularities, but leaving the overall winding-number unchanged.

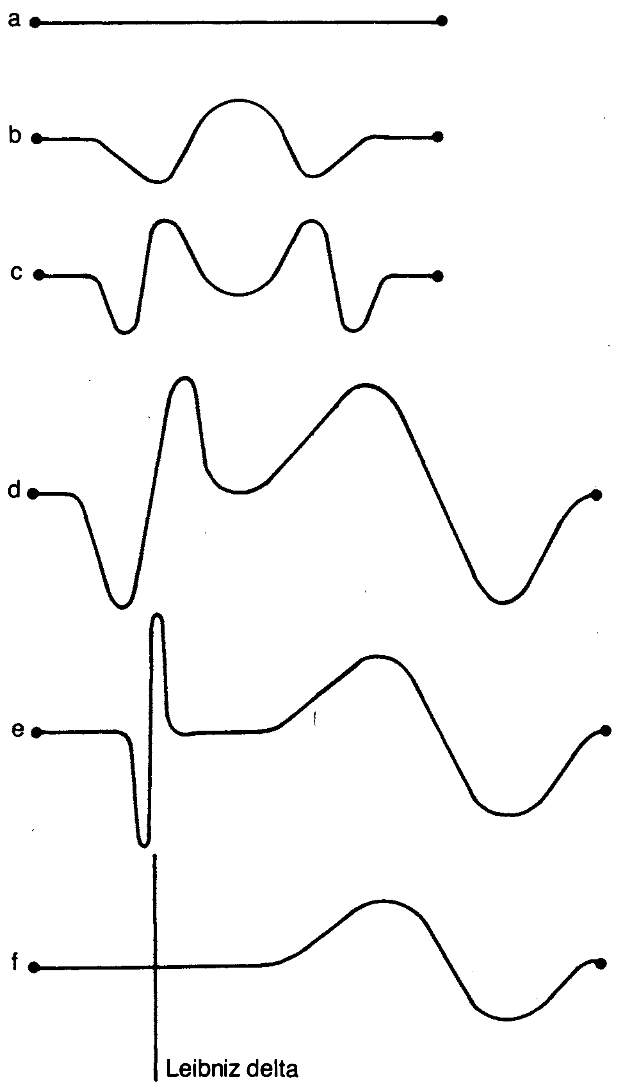
In this way, we can roughly model some of the typical characteristics of quantum phenomena (**Figures 6-a** through **6-f**).

Remarks on this construction

1) In identifying the notion of a *compression* with a shock wave, one must think of the way in which the originally smooth form of the front of a pre-shock-wave is compressed to a point during the course of shock-wave formation (**Figure 7**).

I refer to the construction of the "ruled surface model" of accoustical shock-wave generation published in the Dec. 7,

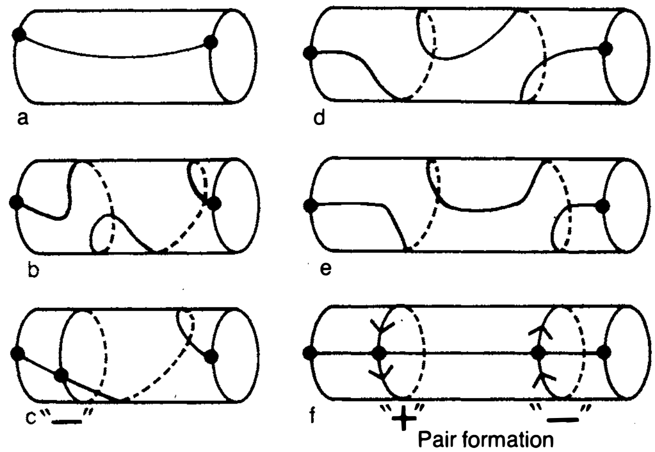
Figure 5



1982 *EIR* [LaRouche, "What is an Economic Shock-Wave?"]. It is easy matter to modify that version of the acoustical shock-wave principle, to obtain a process by which a wave-form: (**a**) would be (**Figure 8**) continuously compressed into a singularity; (**b**) this process would have to be expressed in terms of complex-valued waves, and must be defined so that only the sharply curved loops of a certain type would be compressed, and others maintained. More must be written later on this particular point. It is interesting to note the similarity between the sorts of singularities created by compression in this indicated manner, and the "delta-functions" introduced by Dirac in the mathematical formulation of quantum mechanics.

2) It might be objected, against this way of looking at quantum processes, that the transformation described above

Figure 6



still involves a kind of continuous deformation, whereas real quantum phenemona are presumed to jump from one harmonic configuration to another, with nothing in between. It might also be objected, that in the process of formation of the compression, the form of the wave is distorted, away from the regular form corresponding to a regular polygon. The premises for such objections are undermined once we view the process from a relativistic standpoint.

Assume that the picture of the process, as given above, corresponds to the point of view of an observer "outside" the (visual) universe. This observer sees a continuous deformation and compression of a loop into a circle-singularity. Another observer, viewing the same process from the "inside," sees the matter differently, *insofar as the inside observer depends on the wave itself to define his visual metric*.

We restate the point just made by aid of a simplified example. (**Figure 9**). **An "outside observer" might see the polygon inscribed in a circle as in Figure 9a,** whereas the "inside observer" whose vision is determined such that lines bounded by vertices are straight and of equal length, will "see" the same construction as in **Figure 9b.**

Figure 7

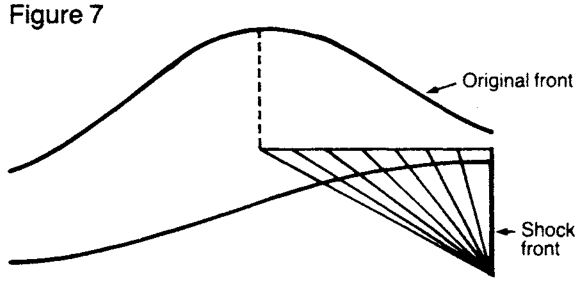


Figure 8

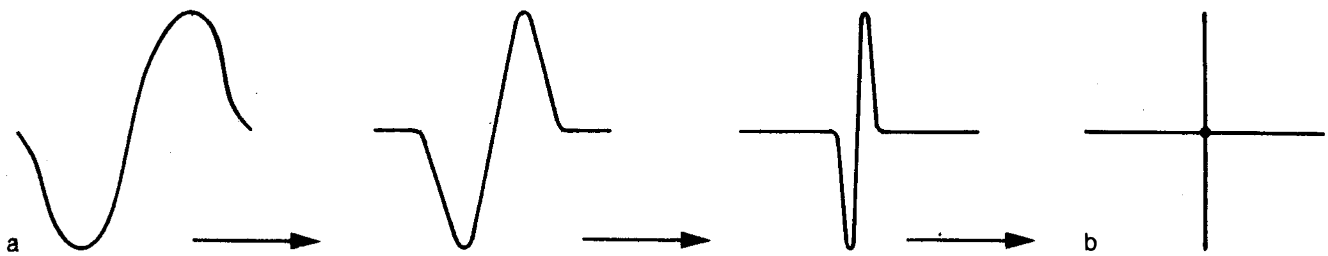
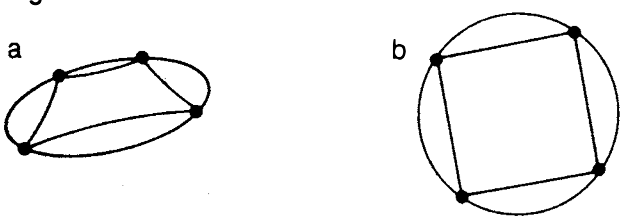


Figure 9



In this illustrative case, the observer on the inside will "see" a completely discontinuous jump from a sine-curve of N oscillations to one of N + 1 oscillations, with nothing in between. Only at the moment of formation of the singularity does the observer on the inside observe a change in terms of *his* metric. The dynamic of the change belongs to a manifold not immediately observable in terms of the inside observer's adopted metric. (The relationship to the Leibniz "delta" may be noted.)

In the body of this report, I show how the basic elements of special relativity theory can be derived from constructions in terms of waves and interference of waves. This, I show, can be accomplished without appeal to an a priori notion of "rigid body," "moving coordinate system," or "clocks." The waves themselves are used to define the "visual" metric in the manifold.

3) The use of *complex* waves in the above construction is crucial. In the terms of real-valued waves, there is no obstacle to continuously deforming a real wave of N cycles into one of N + 1 cycles. The consideration of helical curves, which are complex-wave functions, on a cylinder or torus, on the other hand, brings out an intrinsic topological principle leading to the needed quantization. This is key to the essential way in which complex numbers appear in both the Schrödinger and all other known formulations of quantum mechanics.

4) In the simplest physical case, the hydrogen atom, the de Broglie wavelength of an electron at energy-level E is given by

$$\lambda = \frac{h}{p} = \frac{nh^2}{2\pi m_e e^2}$$

Where M_e is the rest-mass of an electron, and e is the charge. Since the corresponding circumference of the Nth orbit is

$$2\pi p_n = 2\pi \left(\frac{n^2h^2}{4\pi^2 m_e e^2}\right)$$

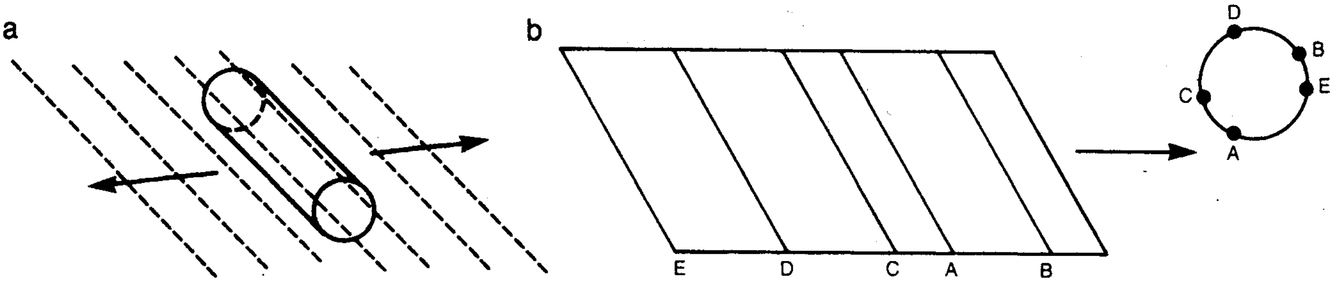
the de Broglie wave divides the orbit circle into N equal parts, i.e., has the winding-number N. In this sense, the quantized energy levels of hydrogen correspond to the regular-polygon divisions of the circle's circumference. We may think of the shock-wave as the action of incoming light on the hydrogen atom, and of the particle accompanying the jump, from level N to N + 1, as representing a captured photon. When the atom jumps back to a lower level, the captured photons are "untied," and re-emitted.

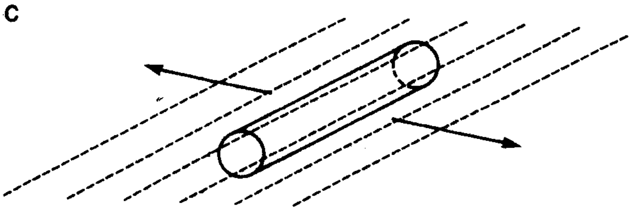
A synthetic-geometrical construction for special relativity theory

Minkowski's formulation of special relativity theory in terms of a hyperbolic metric for 4-space, reduces relativity theory to a simple and beautiful geometrical principle. However, this formulation suffers one characteristic flaw, a flaw which appears mathematically trivial, but which is of fundamental conceptual significance. Although the physical content of special relativity is based on the coherence of all physical phenomena with the propagation of light-waves, Minkowski does not begin with the notion of a wave. Instead of choosing the notion of a wave as the root of his formulation, he constructs his argument from the starting-point of assuming the axiomatic existence of an abstract point in space. That embedded assumption imposes a wave-particle paradox from the start.

In the task-force Lyndon H. LaRouche, Jr. has assembled for elaboration of his analytical method of economic forecasting, the LaRouche-Riemann method, it has become habitual to follow his policy of rejecting any construction which depends upon the hereditary assumption of axiomatic geometry. LaRouche, Uwe von Parpart, et al. have insisted upon the building of all mathematical constructions from the premises of strictest adherence to the method associated with

Figure 10





Jacob Steiner's synthetic geometry.[2] The practical object of this has been to avoid precisely the class of ontological fallacies typified by the cited case from Minkowski.

As Nicholas of Cusa, Steiner, and others have shown, mathematical points and straight lines have no independent, axiomatic existence in fact, but, rather, they are geometrical existences properly derived from the only truly primitive existence in plane geometry, the circle, as the circle is defined by the well-known, elementary topological theorem. Through deriving, and defining the "straight line" and the point from the standpoint of the circle, a pervasive class of formal fallacies is uprooted from mathematical constructions. So, here, I have undertaken a fresh examination of the Minkowski problem from the same standpoint of rigor.

I propose to reconstruct Minkowski space, using the notion of *the wave as elementary*. Interferences of waves, corresponding to folding and intersecting circles, define the world-lines and space-like hyperplanes. The oscillations or "frequency" of the waves determine the metric of time, their wavelengths the metric in space.

As the following, condensed outline of my constructions should indicate, this idea leads to a very simple derivation of the main features of special relativity. This avoids the massive mystification of the subject introduced from the side of Bertrand Russell et al. In this approach, the de Broglie relations, connecting the energy and momentum of a particle with the frequency and wave-length of a corresponding "material wave," come out as an immediate consequence of the construction. This is a happy result, insofar as de Broglie based his original prediction of matter-waves on a simple consideration in relativity theory. This connection, which is stressed in my own approach, has been largely overlooked, despite the fact that de Broglie repeatedly emphasized it in his early writings.

First, I present the geometrical construction, and then discuss its empirical basis. Nothing essential is lost by considering here only one space dimension, rather than three.

I begin with a plane, which represents the space-time manifold, and with an infinite cylinder. Rolling the cylinder over the plane, I define a complex, standing wave. In this way, to each plane locus corresponds a position on a circle, namely, the point at which a circle-of-reference, drawn on the cylinder, touches the plane at the moment the cylinder passed over the locus in question (See **Figure 10, a,b,c**)

In other words, it is the rotation, or complex number, required to rotate the cylinder from a given orientation of reference to the orientation which the cylinder has when passing over the locus. The lines on which the cylinder touches the plane as it rolls are the wave "fronts," the lines of equal phase.

Next, introduce another cylinder, and roll this over the plane in a different direction, generating so a second standing complex wave in spacetime. This defines a second set of parallel wavefronts, which make a constant angle with the lines defined by the first wave.

We proceed as follows to define two sets of loci, the "world-lines" and the space-like lines. The first, the world-lines, are defined as the loci of the intersection of lines of equal phase, as defined by the two cylinders as they roll forward from a given initial position (**Figure 11, a** and **b**).

The second, the space-like lines, are defined as the loci of intersection of the lines of equal phase, defined by the two cylinders, as one rolls forward, the other backward—or vice versa—from any given initial positions (**Figure 12**).

In such a construction, the world-lines correspond to re-enforcement of the two waves, whereas the space-like lines correspond to their cancellation.

The two waves, which coincide along the world-lines, define "time" for those lines. One *oscillation*, corresponding to one rotation of the cylinders, *shall be the unit of time*. On the spacelike lines, *one oscillation* of either of the waves, which have opposite phase along those lines, *defines the unit of length*. Look (**Figure 13**) at an "elementary paral-

Figure 11

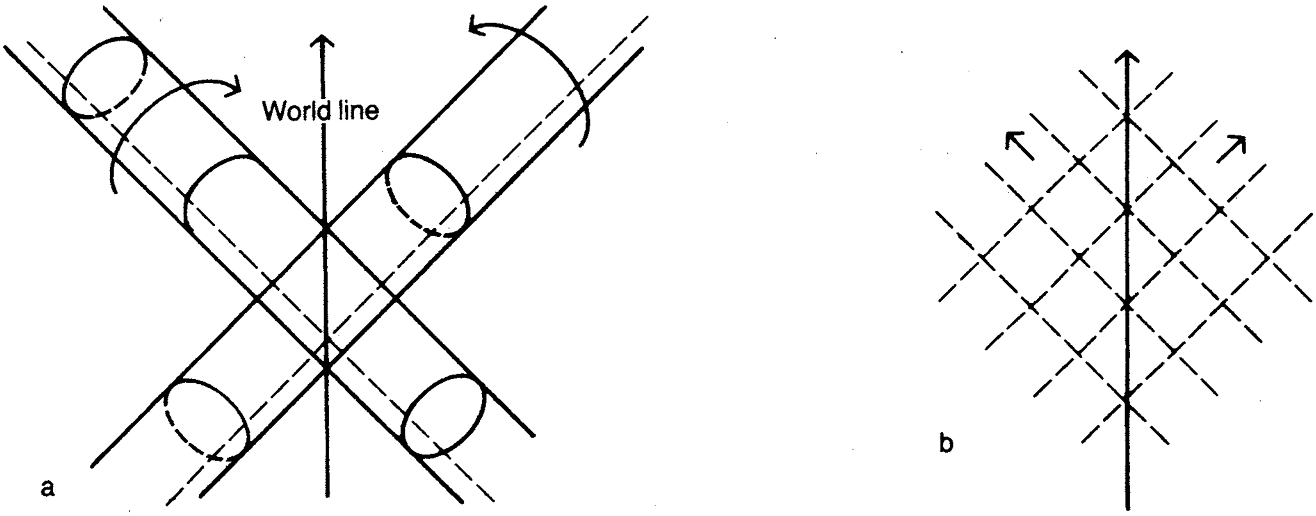


Figure 12

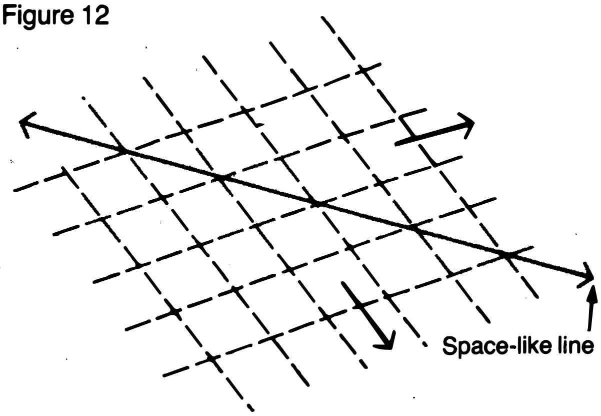
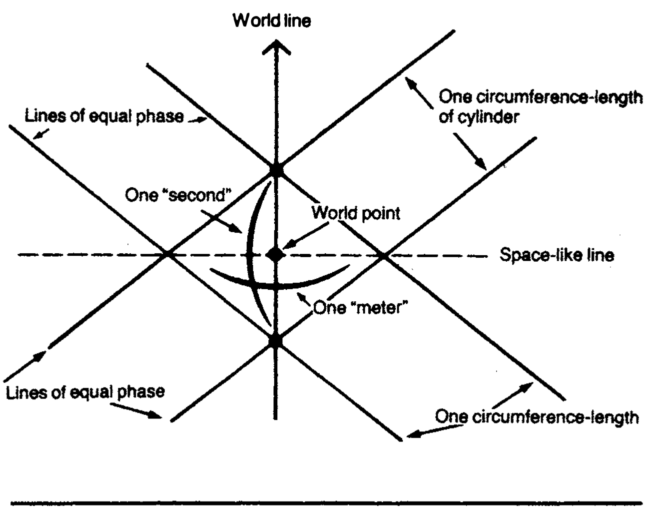


Figure 13



lelogram" defined by rolling each cylinder one turn forward from given positions.

Of the two diagonals, one is a world-line and the other a space-like line. Their intersection defines a world-point, an event at a determined point in space and time. The space-like line corresponds to the world-points regarded as simultaneous with the intersection-point, and the world-line corresponds to the future and past for the "place" defined by the world-point.

Following all the world-points on the space-like line together, i.e., translating that space-line in the direction of the world-line, we find that the two waves propagate along the space-like line in the opposite direction, with frequency 1 and wave-length 1 (**Figure 14**).

So far, we have defined everything in terms of the two initial waves. Next, we take two other cylinders. The sizes of these additional cylinders are determined so that the cir-

Figure 14

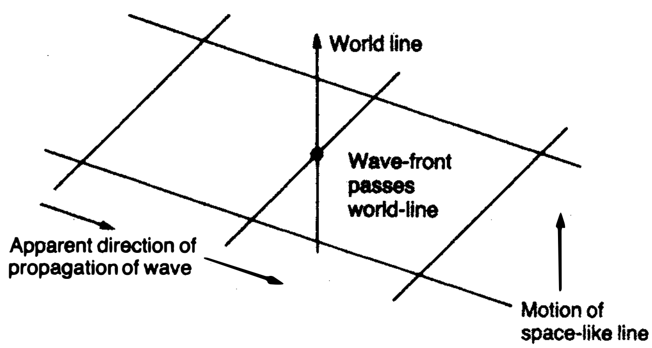
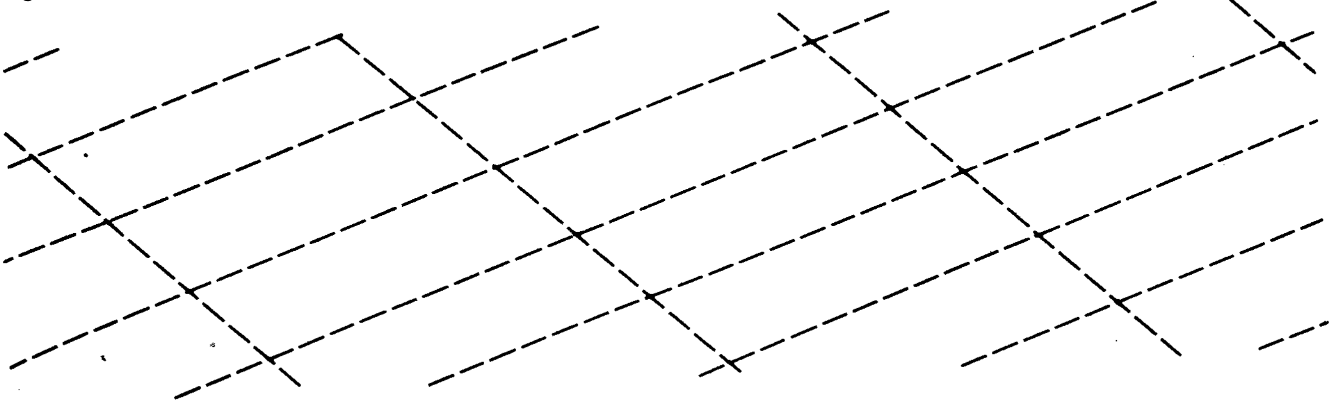


Figure 15



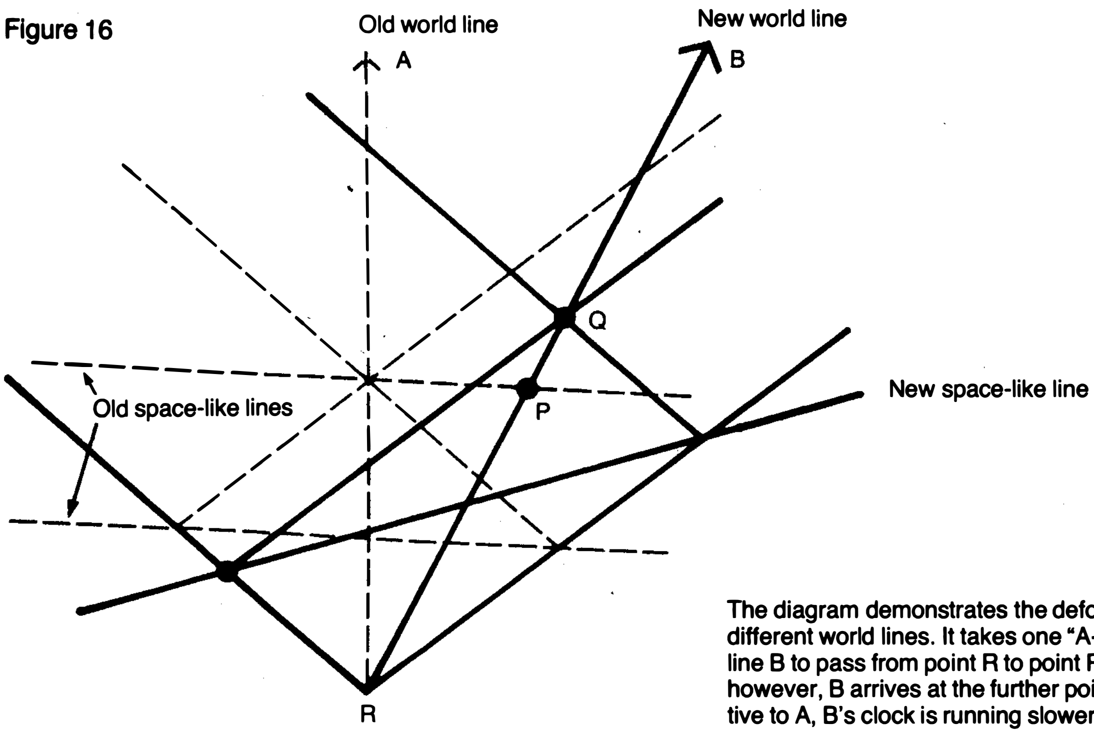
cumference of the first new cylinder is smaller (or, respectively, larger) than the first of the old pair of cylinders in the same proportion that the circumference of the second old cylinder is smaller (or, respectively larger) than that of the second new cylinder. In other words, the sizes of the new cylinders are proportioned such that the product of their circumferences is equal to the product of the circumferences of the old cylinders. In other words, the areas of the new elementary parallelogram generated with the new cylinders will be equal to the area of the elementary parallelogram generated by the old.

We repeat the previous constructions with the new cylinders, rolling them in the same directions as the old. We obtain in this way new complex waves, whose fronts are parallel with the old, *but in this new case the wave-lengths are changed* (**Figure 15**).

The new waves generate a new set of world-lines and space-like lines, with space and time metrics so defined that the new waves have frequency and wave-length values of "one" *in the new system.*

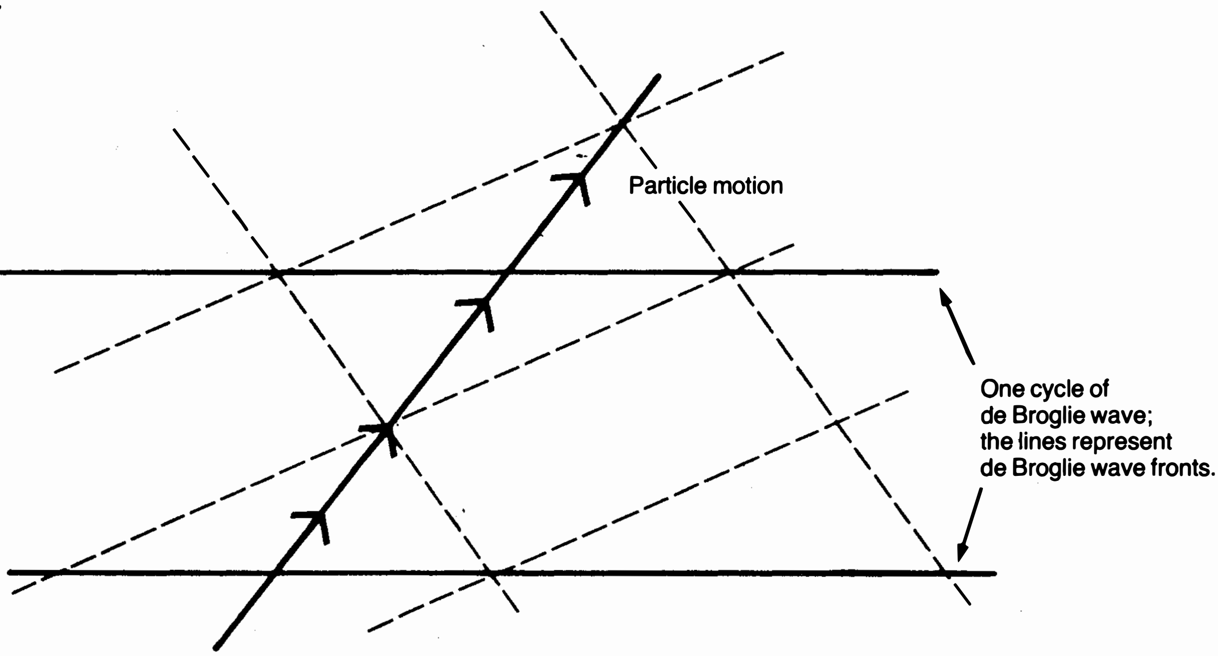
It is now a simple matter to compare these two systems, the new and the old, to the effect of obtaining the familiar

Figure 16



The diagram demonstrates the deformation of time for different world lines. It takes one "A-second" for world line B to pass from point R to point P; in a "B-second," however, B arrives at the further point Q. Hence, relative to A, B's clock is running slower.

Figure 17



Lorentz transformation, Doppler effect, and so forth. Visualize this with aid of the two elementary parallelograms (**Figure 16**).

This diagram demonstrates the deformation of time for different world-lines. One "A-second" is required for world-line B to pass from point R to point P. However, in a "B-second," B arrives at the further point Q. Hence, relative to A, B's clock is running slower.

De Broglie's wave-particle correspondence is a simple matter to define in these terms of reference.

If world-line B, above, represents the motion of a particle of normalized mass 1, then the corresponding system of space-like lines represents the wave-fronts of de Broglie's wave for that particle. The frequency and wave-length of the de Broglie wave along any arbitrary world-line and space-like line are determined by the prescription that the de Broglie wave has frequency 1 along the world-line of the particle (**Figure 17**).

This was de Broglie's original idea. The particle is thought of as a localized oscillatory phenomenon, and the de Broglie wave is chosen to be constantly in phase with the particle-oscillator.

In empirical terms of reference, the foregoing construction can be restated as follows.

Given two light-waves of arbitrary frequency, propagating along a line in opposite directions, the corresponding world-line is defined by the motion of an observer for whom the waves appear *to have the same color*. Assume this color were yellow, for example. In shifting to the new set of waves, the observer sees a shift in one direction toward red, in the other direction toward blue.

The new observer, defining the new world-lines, must move relative to the old observer in such a manner as to see again equal colors. For reason of the proportioning of frequencies we prescribed by our construction above, the equal color must be yellow.

The light vibrations themselves define the "clocks" for these observers. The apparent wave-lengths of the light, as determined by interference measurements made by the observers, define their "measuring rods." *The constancy of the speed of light is not a law of the universe, but is a product of the manner in which the metric is defined.*

How much is mathematical tautology, and how much empirical fact? It is true of all oscillating systems, and especially so of limited systems which return periodically to their initial configuration, that these systems remain "in tune" with the light-clock. It is the case, that all rigid bodies, such as crystals, behave as if their spacings were determined by interference patterns of light. This is equivalent to stating, that all physical systems cohere with the harmonic ordering of space-time, and that the harmonic ordering of space-time is expressed by the propagation of light.

[1]Lyndon H. LaRouche, Jr., "What Is An Economic Shock-Wave," *EIR*, Vol. 9 Nos. 47-48, Dec. 7,14, 1982.

[2]See, for example, Jacob Steiner, *Geometrical Constructions with a Ruler Given a Fixed Circle with its Center*, Scripta Mathematica, Yeshiva University, New York, 1950.