The Importance of Riemann Today

by Bruce Director

April 23—Bernhard Riemann's habilitation dissertation—see *The Importance of Bernhard Riemann* by Bruce Director, *EIR* March 25, 2016—is the most famous expression, and most accessible to the non-scientist, of Riemann's revolutionary discoveries. But citizens wishing to understand and act on the crucial matters of politics, economics, and science that will determine whether Mankind survives the current crisis, would be well advised to acquaint themselves with the broader scope of Riemann's work.

In doing so, you will find many wonderful results that have laid the foundation for virtually every important development in science for the last century and a half, but even more important, an insight into a creative thinker who recognized, in his own creativity itself, the principles on which the organization of the universe is based. It is that quality of thinking that the world is in such need today.

I will give several examples to illustrate the point just made, beginning with Riemann's earliest published work.

By the time Riemann came to Göttingen to study with Gauss in 1846, he had already concluded that any new discovery in science must come from rooting out the stultifying method of thinking that had become dominant in Germany since the rise of Immanuel Kant. Gauss had already recognized this and in his early years took it on quite aggressively, but after the rise of Napoleon and the subsequent reaction, he had kept much of his thinking under wraps.

Kant had reintroduced Aristotle's separation of mind from the universe as a reaction against the great achievements of Gottfried Leibniz, in an attempt to seal off science from creative thinking. According to Kant's dictum, pure thinking could only proceed by a set of rules abstracted from all reality outside the mind. Hence, protected from the unruly world of material things and the unreliable world of sense perception, a system of pure reason could be constructed that was reliable.

The problem was that such a system was as impotent as it was useless. This didn't bother Kant, who developed a system of practical reason and other compromises to deal with the real world, as long as the world was made orderly by a controlling oligarchy (either human or deified). It nevertheless served to put a constraint on creative thinking in science, art and politics, which, fortunately, was disrupted by Prometheans such as Wolfgang Amadeus Mozart, Ludwig van Beethoven, Friedrich Schiller, Franz Schubert, Abraham Gotthelf Kästner, and Karl Friedrich Gauss.

From Leibniz to Riemann

The seemingly most secure refuge for Kantianism was pure mathematics, and within that domain, algebra and number theory,— as these, Kant insisted, were creations of pure reason, and could not be polluted by the unmanageable world of matter and mind. But lurking in this world of pure logic was an unwelcome spirit, the square root of –1, that had so bedeviled the inner sanctum of pure reason that it had earned for itself the appellation, *impossible* or *imaginary*.[1]

The reality was that the square root of –1 isn't impossible. It shows up repeatedly in the system of algebra or number.[2] It was only "impossible" because its meaning was in the real world, not the abstract world of pure reason. Gauss insisted that like negative numbers, the concept of "imaginary" numbers was not derived by

1. Denoted by the letter *i*.

2. For example, the abstract algebraic expression $x^2–y^2$ can be factored into (x+y)(x–y). But the expression of the physically real Pythagorean theorem's x^2+y^2 can only be factored algebraically as $(x+iy)(x–iy)$. Similarly Gauss showed that prime numbers, the seeming bedrock of all counting numbers, are dependent on impossible numbers, as for example, in the case of all 4n+1 that are primes, such as 5. Such numbers are not really prime, as they can be factored, such as $5=(2+i)(2–i)$.

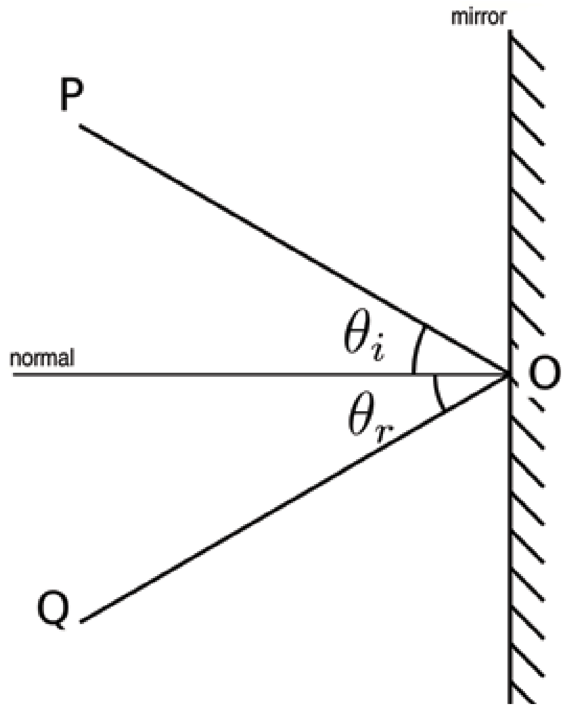
completing the formal rules of arithmetic, but rather by the physical process of direction which, he emphasized in contradistinction to Kant, could not be derived by "pure reason."[3] In his own doctoral dissertation on the fundamental theorem of algebra, Gauss had demonstrated this, which had caused quite a stir when it was issued. But even though his notebooks were filled with many developments on the subject of what had become known as complex numbers, his published work on the subject was almost nothing.

Into this environment came Riemann, who sought out Gauss as a doctoral advisor. In 1851, Gauss supervised Riemann's revolutionary dissertation on "Functions of a Complex Variable." Though the work is most often falsely relegated to the domain of pure mathematics, anyone who has studied Gauss and Riemann knows that that is not true. In fact, in his dissertation and his other works on Abelian Functions and Hypergeometric Functions, Riemann laid down a method of physical thinking that uncovered the connection between the way the mind works, and the physical universe works, and that it was only by gaining a deeper understanding of the former, that science could hope to grasp anything meaningful about the latter.

The core of Riemann's thinking is rooted in Leibniz's ideas of *least action* and *analysis situs.* Leibniz had insisted, in opposition to Descartes and the pragmatists of his time, that nothing irrational could happen in the universe, as that would render the human mind irrelevant. Consequently, the universe must be governed by principles that were not directly observable by the senses, but were nevertheless knowable by the human mind. One such concept is the principle of least action.

This is best illustrated pedagogically by an example. When light strikes a mirror, it is reflected at the same angle that it strikes the mirror (**Figure 1**). This is an observation verifiable in the domain of sense perception. But sense perception is incapable of answering the question, why does light act in this way? The formulation of the question, and its answer, is an act of mind acting in and on the universe. Ancient scientists had already recognized that the equality of the angle of incidence with the angle of reflection meant that the overall path of the light was the shortest possible distance. Is this a particular characteristic of light, or only a special case applicable to this particular phenomenon?

FIGURE 1



When light travels through two different media, such as air and water, the angle of incidence and the angle of refraction are not equal[4] and, consequently, the path of the light is no longer the shortest distance (**Figure 2**). Is this a violation of the principle observed in the case of reflection?

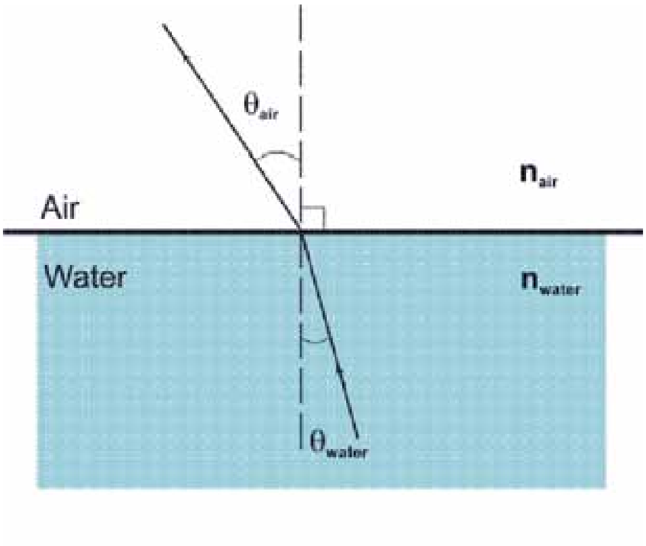
Pierre de Fermat (1601-65) showed that the behavior of light under refraction, did not actually violate the principle of shortest path observed in reflection, but, rather, it reflected a higher concept of "path." Since the light changed its speed between the media, the shortest path had to be understood as the path of *least time*.

Leibniz saw this behavior of light as a reflection of a more universal concept that he called the "principle of least action," which, he emphasized, reflected the functional congruence of the creative powers of the human mind with the organization of the universe itself.

3. See Gauss' "Second Treatise on Biquadratic Residues" cited by Riemann in his habilitation dissertation. Gauss noted that the notion of positive and negative numbers indicated magnitudes situated in opposite directions, and that "imaginary" numbers indicated magnitudes orthogonal to the "real" numbers. He hypothesized the existence of a third set of numbers orthogonal to the real and imaginary, but never developed the idea. Riemann showed that this was unnecessary.

4. Though the angles of incidence and refraction are not equal, the sines of these angles are in constant proportion.

FIGURE 2



$n_{air} < n_{water}$

From this foundation, Riemann created an entirely new way of thinking. He conceived of the idea that the investigation of physics must be centered, not on the phenomena, but on physical manifolds. Over the course of many works, Riemann developed the notion of a physical manifold as a single conception under which a multiplicity of physical principles act. It is the nature of the manifold, accessible to the mind, but not sense perception, that determines the phenomena under investigation. Hence, abstract geometries and formal mathematical structures are discarded as useless.

Physical Manifolds

For example, from Riemann's standpoint, the difference between the behavior of reflected and refracted light, is not an effect of two different phenomena, but the same phenomenon acting in manifolds of different degrees of action. Reflection takes place in a manifold in which only direction changes, whereas refraction occurs in a manifold in which both direction and speed change.[5] What didn't change is the governing principle of least action. The expression of least action is, thus, conceived as a function of the characteristics of the manifold.

Riemann discovered that the domain of the complex numbers, having their origin in the interaction between the mind and nature, was uniquely suited to express the essential characteristics of a physical manifold. He called the principle at work here the "Dirichlet"[6] principle, which said that any bounded manifold expressed the principle of least action in a unique way. Riemann realized that this is expressed by a system of curves of minimum and maximum curvature which were always orthogonal to each other.[7] Because orthogonality is a physical expression of complex numbers, functions of a complex variable are uniquely suited to express the least action principle in physical manifolds.

This became the basis for Riemann to develop a general theory of manifolds in which he showed that only a small number of parameters, specifically the boundary conditions and number of singularities, determined the characteristic paths of least action.

Riemann went still further. The scientist, like the statesman or military leader, must discover the characteristics of a manifold as an active participant in the action. Friedrich Schiller described this in terms of creating political freedom as akin to fixing the mechanism of a clock while the clock is still running. This requires being able to discern the global characteristics of a manifold from its infinitesimal action.

Such an approach was not new to Riemann. Johannes Kepler had accomplished his results by determining the general characteristics of the solar system from a moving planet within it, by recognizing changes in the infinitesimally small. Thus every small part of a planet's motion reflected the overall characteristics of what Kepler understood as the solar system as a whole. We now know that the effects of galactic and intergalactic processes are also at work here. This was recognized by Riemann who, in his habilitation dissertation, emphasized that science must look into the very large and very small to understand nature.

Kepler's approach was further developed into the infinitesimal calculus by Leibniz, who formulated a more general approach that he called *analysis situs.* Gauss, in his investigations into geodesy and terrestrial magnetism, extended Leibniz's method, showing that such global characteristics as curvature and shortest

5. The former being a manifold of space, while the latter is a manifold of space-time.

6. Named for Lejeune Dirichlet, his predecessor at Göttingen. Dirichlet had been a protégé of Gauss and Alexander von Humboldt. Riemann had studied with Dirichlet for a year in Berlin. As the husband of Rebecca Mendelssohn, Dirichlet was involved in organizing collaboration among musicians and scientists when he came to Göttingen. Riemann participated in these collaborations.

7. Riemann's work here is a generalization of Gauss's concept of potential.

path (geodetic) could be determined from infinitesimally small measurements. For example, Gauss was the first person to determine the characteristics of the Earth's magnetic field, and the location of the south magnetic pole, purely from a careful *analysis situs* of small local variations in the Earth's magnetic field.

But Riemann took this even further. Elaborating a theory of functions of a complex variable, Riemann created a means by which the essential physical characteristics of a general manifold could be known from very small measurements, thus restoring the primacy of concepts over calculations in science. For anyone wishing to provide leadership in the domain of politics or economics today, a thorough grasp of Riemann's method is essential.

The elaboration of Riemann's theory of complex functions gives us a sense of Riemann's conceptual approach. But he was not a mere theorist. Riemann applied this method to some of the most outstanding problems in physics of his time, in the fields of electromagnetism, hydrodynamics, and geodesy. His efforts in these areas of applied physics repeatedly led to discoveries that showed that the reductionist methods which were in widespread use at the time, were not only conceptually inferior, but also produced wrong results.

Physics and Life

One of the best examples is Riemann's work on what has become known as shock-waves. In a compressible medium such as air, sound waves appear as alternating regions of compression and decompression of the air. It is a well-known observation that such waves propagate at a finite speed that is independent of the frequency (perceived as pitch) or the amplitude (perceived as volume) of the wave. From the above description and the mathematical analysis of a wave function, it would appear that this finite speed of sound is a limiting velocity that can not be surpassed. Riemann, however, saw it completely differently. He realized that if the alternating regions of compression and decompression overtook each other, a new state of organization would come into existence, creating a new structure that would propagate through the air at its own speed as if it were itself a material object. Today such structures are commonly known as "shock-waves." Riemann's hypothesis concerning shock-waves was considered ludicrous by the experts at the time. After his death, the experimental demonstration of shock-waves proved him right and his detractors short-sighted. Once again, Riemann showed that mind, not mathematical formalism, reflects the world.

Toward the end of his life, Riemann began to investigate his long-held conviction that progress in science could only take place if the boundary between abiotic physics and living organisms were superseded. In his last work, Riemann presented his research into the mechanism of the human hearing apparatus. Analyzed as an abiotic mechanism, as Hermann von Helmholtz had done, human beings perceive sound when the compression waves of the air impact the tympanic membrane (eardrum), which in turn activates three small bones in the middle ear (anvil, hammer, and stirrup), which in turn set into motion a wave in the fluid in the inner ear, which then vibrates small hair fibers that translate the vibrations into electrical impulses which are perceived by the brain as sound.

The above approach attempts to explain the action of a living organism as if it were a collection of abiotic physical machines. Riemann noted that this was patently absurd. Were Helmholtz's theory true, then human beings could not be able to perceive the very subtle variations in timbre, volume, pitch, and nuance that make possible the discernment of language and polyphonic music. Though Riemann died before he could further elaborate an approach to understanding hearing, his study posits the exciting and provocative idea that the investigation of all physical processes must be subsumed by the higher concept of life. In this way, Riemann laid the foundations for the breakthroughs in this direction by Vladimir Vernadsky, and set the stage for new areas of science that are yet to be explored.

This approach did not come late in life to Riemann. When his papers were compiled after his death, a series of fragments on mind, life, and philosophy were discovered that give us an insight into the source of his remarkable ability to see far beyond the appearances. These works, published posthumously as his Philosophical Fragments, show that all of Riemann's thinking about physics started with a deep appreciation of the creative powers of the mind. It is his concept of how the mind works that is reflected in his thinking on how the physical world functions.

It is best to let Riemann speak for himself:

> With each simple act of thinking, something durable, substantial enters our mind. This substance appears to us, in fact, as a unity, but it appears (in-

> sofar as it is the expression of space and time extension) as comprising a subsumed manifold; I name this a *thought-mass.* To this effect, all thinking is the development of new thought-masses. The thought-masses entering into the mind appear to us to be images; their varying internal states determine how they differ qualitatively.
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> As they are forming, the thought-masses blend; or are folded together, or connect to one another and also to older thought-masses, in a precisely determined manner. The character and strength of these connections depend upon causes which were only partially recognized by Johann Friedrich Herbart, but which I shall fill out in what follows. They rest primarily on the internal relationships among the thought-masses.
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> The mind is a compact, multiply connected thought-mass with internal connections of the most intimate kind. It grows continuously as new thought-masses enter it, and this is the means by which it continues to develop. Thought-masses once formed, are imperishable; and their connections cannot be dissolved; only the relative strength of these connections is altered by the addition of new thought-masses.
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> Thought-masses need no material carrier for their continued existence, and exert no lasting effect upon the physical world. Therefore they are not related to any portion of matter, and have no position in space. On the other hand, a material carrier is required for every entry, generation, every formation of new thought-masses, and for their unification. Thus all thinking does occur at a definite place.

In other fragments, Riemann noted that this process of concept formation was inherently social, transmitted through culture and language within and across generations. He further indicated that the development of ideas in human beings is the highest expression of a universal process that encompasses the living and non-living domains.

For its continued survival, the human race desperately needs a revival of scientific thinking of the quality of Riemann. A first step would be to rediscover what Riemann actually did and thought, which is something of which almost no one alive today, except Lyndon LaRouche, has much of an understanding.