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Riemann Refutes Euler: Behind an Earthshaking Scientific Discovery

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Editors' note: *The magazine **21st Century Science & Technology** published, in its Winter 1995 edition, an English translation of a collection of early writings of Bernhard Riemann. We publish here Lyndon LaRouche's introduction, "Riemann Refutes Euler," by permission of **21st Century**.*

In the following pages, **21st Century** presents the first known publication in English translation, of a group of posthumously published early writings of the famous physicist Bernhard Riemann (1826-1866).[1] These have the special significance of providing some relatively indispensable background for understanding how Riemann came to develop his earthshaking discoveries of 1853-1854.[2]

The special relevance of these pieces, pertains to the fact, that there can be no competent appraisal of Riemann's work, which does not treat his writings as, like those of Karl Weierstrass, a devastating refutation of Leonhard Euler's savage attacks on Gottfried Leibniz.[3] The formal issue is the question, cloaked in a discussion of mathematical series, whether or not mathematical discontinuities exist.[4] The relevant substantive issue behind these attacks on Leibniz by the Eighteenth-Century newtonians, Dr. Samuel Clarke and Leonhard Euler, is, much more today than during Riemann's time,

1. See **Bernhard Riemann's Gesammelte Mathematische Werke**, Heinrich Weber, ed. (New York: Dover Publications reprint, 1953), "*Fragmente philosophischen Inhalts,*" pp. 507-538. A more recent reprint of the same, Heinrich Weber's second edition (Stuttgart: B.G. Teubner, 1902), is Vaduz, Liechtenstein: Saendig Reprint Verlag Hans R. Wohlwend. Hereinafter, this is identified as **Riemann Werke**.

2. See Bernhard Riemann, "*Über die Hypothesen, welche der Geometrie zu Grunde liegen*" ("On the Hypotheses Which Underlie Geometry"), **Riemann Werke**, pp. 272-287. This is the famous June 10, 1854 habilitation dissertation, to which Albert Einstein referred, in identifying Riemann's work as a root of General Relativity. On the dating of the work embodied in this dissertation, 1853-1854, see H. Weber's reference to Riemann's note, which dates the discovery underlying the paper to "March 1, 1853": **Werke**, p. 508.

3. On Euler's attack on Leibniz, see Lyndon H. LaRouche, Jr., **The Science of Christian Economy** (Washington: Schiller Institute, 1991), Appendix XI, "Euler's Fallacies on the Subjects of Infinite Divisibility and Leibniz's Monads," pp. 407-425. That appendix includes the sections of Euler's **Letters to a German Princess** (dated by him May 5, 1761) in which his second explicit attack on Leibniz is made. The first occurred as his role in the scandalous case of Pierre-Louis Maupertuis, whose exposed fraud on the subject of "least action" led to Maupertuis's 1753 ouster from direction of the Berlin Academy; Euler was the principal accomplice of Maupertuis in perpetrating that hoax. We emphasize the primary coincidence between Riemann and Weierstrass here, not their secondary differences in approach.

4. See Leibniz-Clarke correspondence on the subject of the relationship between infinite series and the differential calculus. (G.W. Leibniz, **Philosophical Papers and Letters,** edited by Leroy E. Loemker, 2nd edition [Dordrecht: D. Reidel, 1969, reprinted Boston: Kluwer Academic, 1989], pp. 675-721.) Although Leibniz's development of the differential calculus had roots in some of his earlier activities, the archival evidence is, that what became known as Leibniz's calculus was actually developed during 1672-1676, in Paris, at Jean-Baptiste Colbert's Royal Academy of Science. Leibniz's first paper, presenting the discovery, was submitted for publication, in Paris, in 1676, immediately prior to his return to Germany. Isaac Newton's international reputation, and the Newton-Clarke attack on Leibniz, was created by Venice's Paris-based Abbot Antonio Conti (1677-1749), who sponsored a network of salons throughout Europe, a network devoted to the principal mission of seeking to discredit Leibniz, and build up Newton's reputation. Dr. Samuel Clarke was an agent of Conti, as were the Berlin circles of Maupertuis and Euler.

Bernhard Riemann (above) and Leonhard Euler (right). "Like Leibniz before him, Riemann's discovery demonstrates that formal mathematical-physics schemes do not embody the potentiality of a truth-doctrine. To find truth, we must depart the domain of mathematics, and go over into another domain, the realm of experimental physics."

whether physics is a branch of mathematics, or mathematics a branch of physics.

As in the concluding sentence of his famous 1854 habilitation dissertation, Riemann demonstrated that, to settle the underlying issues of mathematics, one must depart that domain, into physics.[5] That statement plants Riemann, like his sponsor Karl Gauss before him, fully within the domain of physics, rather than the virtual reality which one associates with the influence of Bertrand Russell and the Bourbaki *Golem* upon much of today's teaching of mathematics. The posthumously published papers presented in English translation here, bear directly on Riemann's development of his approach to that issue.

Riemann and Economics

21st Century's attention to Riemann reflects my own original work in a branch of physical science founded by Leibniz, known as physical economy. My discoveries in this field supplied the principal impetus for the mid-1970s founding of the Fusion Energy Foundation, which ricocheted into the later founding of **21st Century** magazine. Although the principal part of my discoveries were not prompted by Riemann's work, the approach adopted for solving the mathematical problems posed by those discoveries was prompted almost entirely by Riemann's habilitation dissertation, leading to the designation of "LaRouche-Riemann Method."[6]

To introduce Riemann's posthumously published papers, I indicate the features of his dissertation which are most relevant to the problems of physical economy. To that end, consider, first, the place which mathematical discontinuities occupy in Riemann's discovery, and then, the significance of Riemann's emphasis on what he terms *Geistesmassen* in the posthumously published papers.

5. "*Es führt dies hinüber in das Gebiet einer andern Wissenschaft, in das Gebiet der Physik, welches wohl die Natur der heutigen Veranlassung nicht zu betreten erlaubt.*" ("This leads into the domain of another science, the realm of physics, which the nature of today's occasion does not permit us to enter.") Habilitation dissertation, **Riemann Werke,** p. 286.

6. See Lyndon H. LaRouche, Jr., "Why Most Nobel Prize Economists Are Quacks," *Executive Intelligence Review,* July 28, 1995, and Lyndon H. LaRouche, Jr., "Non-Newtonian Mathematics for Economists," **Executive Intelligence Review**, Aug. 11, 1995.

First, to define the significance of mathematical discontinuities, I restate Riemann's point of departure in his dissertation in my own words.

The origin of modern mathematics lies in what is commonly identified as a "Euclidean" notion of simple space-time. This idea of space-time pretends to represent the real universe, which it does not represent. It is an idea which is not a creation of the senses, but, rather, of the naive imagination. We merely imagine that space is defined by three senses of direction (backward-forward, up-down, side-to-side), and imagine that these might be extended without limit, and in perfectly uninterrupted continuity. We imagine that time is a single, limitless dimension of perfect continuity: backward-forward. Taken together, these presumptions of the imagination define a four-dimensional space-time manifold, or, in other words, a quadruply-extended space-time manifold.

The naive imagination attempts to locate perceptible bodies and their motions within such a quadruply extended manifold. It may be said fairly, that our imaginary space-time manifold is used as a kind of mental mirror, upon which we attempt to project reflections of motion of bodies in space-time. The result of such projections is a simple "Euclidean" sort of algebraic mathematics, which, we soon discover, is not a mathematics of the real universe.

Classical experiments, typified by the measurement of the curvature of the Earth's surface by the ancient Eratosthenes of Plato's Academy at Athens,[7] supply measurable demonstration that the motion of bodies in physical space-time does not correspond to what a naive, algebraic notion of space-time suggests. We must add non-space-time "dimensions," such as the notions of "mass," "charge," and so forth, to derive a mathematics which agrees with our measurement of the motions which are reflected, from physical space-time, upon that imaginary mirror known as simple space-time.[8]

Thus, in place of a four-dimensional space-time of the imagination, the attempt to explore physical space-time presents us with a physical-space-time manifold of many more dimensions than the four dimensions of naive space-time. We call these added factors "dimensions," because they can be scaled, according to the ordering-principle of "greater than" and "less than," as we do the dimensions of naive space-time. Instead of saying n+4 dimensions, we include the four in our count of n; we speak, thus, of a "physical-space-time manifold of n dimensions." Then, commonly, we attempt to portray motion within that physical-space-time, of n dimensions, in terms of its imaginary reflection upon a four-fold space-time.

In each case, the addition of a validatable new "dimension" to the physical-space-time manifold of reference, corresponds to a change in measurement, a change in the yardstick we must employ to measure the relevant motion, or analogous form of action. For example, Eratosthenes estimated that the Earth was a spheroid of about 7850 miles, from pole to pole (not a bad estimate for the time).[9] This meant, that to measure motion along the surface of the Earth, we must use a yardstick of spherical trigonometry, rather than one appropriate to a simple Euclidean plane. Similarly, once Ole Rømer had demonstrated, in 1676, that the radiation of light was governed by a principle of retarded potential, Christiaan Huygens, in 1677, generalized principles of reflection and refraction accordingly,[10] and, Jean Bernoulli and Leibniz demonstrated that the mathematics of the transcendental domain's special relativity must supersede the algebraic methods of Galileo, Descartes, and Newton.[11]

The validation of the necessary addition of such an added physical dimension, by measurement, implies the challenge to be considered here. Each such addition signifies, that instead of an n-fold physical-space-time manifold, n is superseded by (n+1). This gives us a generalized term of topology, which we might express symbolically by (n+1)/n. The series of changes, from n to n+1 dimensions, is associated with a series of changes in the choice of the yardstick which we must employ to measure the relevant physical action.[12]

7. See "How Eratosthenes Measured the Unseen" (Figure 2), in Lyndon H. LaRouche, Jr., "Kenneth Arrow Runs Out of Ideas, But Not Words," **21st Century,** Fall 1995, pp. 34-53.

8. This image is an accurate representation of the intent of Plato's reference to shadows which reality casts upon the imagination, as if these shadows were reflections on the wall of a cave's firelit interior.

9. **Greek Mathematical Works**, Ivor Thomas, trans., 2 vols. (Cambridge, Mass.: Harvard University Press, 1980), Vol. II, p. 273, note c.

10. Christiaan Huygens, **A Treatise on Light** (New York: Dover Publications reprint, 1962).

11. The "brachystochrone problem": Jean Bernoulli (1696). The equivalence of least time to least action.

12. This does not justify the presumptions of some popularized notions of a differential geometry. The basis for that word of warning will be made clearer below.

This is also the problem which confronts us, in physical economy, as one may attempt to define the correspondence between scientific and technological progress, on the one side, and, on the other side, a general, resulting increase in the productive powers of labor, per capita, per household, and per square kilometer. For that case, the type of yardstick used is termed *potential relative population-density*; that yardstick changes its scale (per capita, per square kilometer) as the level of applied scientific and technological progress advances.

Science and Metaphor

All of the issues posed by Riemann's habilitation dissertation, while most profound, are so elementary that they might be understood at the level of a good secondary school's graduate. Once we accept his intention in that location, that paper is among the most lucid pieces of prose ever supplied to the literature of fundamental scientific discoveries. Admittedly, most of the classroom's putatively authoritative commentators have conveyed a contrary, confused view of this work. The failure of all such commentaries examined, is that the commentators, by refusing to accept the fact of what Riemann is saying, project upon him an intention which is axiomatically contrary to his own.

The axiomatic failures of such authoritative commentators occur on two levels.

Closer to the surface, they have sought to defend such post-1815 authorities in taught mathematics as Newton, Euler, Augustin Cauchy, et al. from the devastating refutation provided by Riemann's discovery. This centers around Euler's argument against Leibniz. That relatively more superficial axiomatic assertion, is the hysterical insistence of the positivists, that, ultimately, mathematical discontinuities do not exist.[13]

On the deeper level, there is a more devastating issue, which the opponents of Leibniz and Riemann refuse to debate.

The radical positivists of the Bourbaki cult exemplify this deeper issue. The peculiar, Ockhamite deism of such positivist ideologues, is the dogma, that all questions of science must be settled by mathematical proofs delivered upon a blackboard, or, by a modern digital-computer system. Every demonstration that mathematical formalism is not the god of science, whether by Plato and his academy after him, or from moderns such as Leibniz or Riemann, fills such positivists with an obscene, irrationalist rage, akin in spirit and rationality to that of Marat's or Danton's Jacobin mob.

This deeper of the two levels of axiomatic issues, underlies the assignment of Abbot Antonio Conti's agent, Dr. Samuel Clarke, for the attacks upon Leibniz. This is the issue underlying the savage, posthumous attacks upon Leibniz by the Conti salon's Euler. This was also the basis for the hyena-like attack, led by the devotees of Ernst Mach, upon Max Planck, during the period of World War I.[14]

Once we acknowledge the primary historical fact of mathematical-physical knowledge, that each of those discoveries of physical principle which is validated by the appropriate measurement, presents mathematics with a topological challenge of the indicated $(n+1)/n$ form, mathematical formalism is stripped of that attributed, god-like authority which the devotees of Euler and the Bourbaki cult defend so fanatically.[15] Like Leibniz before him, Riemann's discovery demonstrates that formal mathematical-physics schemes do not embody the potentiality of a truth-doctrine. To find truth, we must depart the domain of mathematics, and go over into another domain, the realm of experimental physics.

The key to all among these, and derived formal issues of mathematical physics, is the connection between the erroneous insistence, that, ultimately, no discontinuities exist in mathematics, and the deeper assumption (also false), as among the followers of the Bourbaki dogma, that mathematics can be a truth-doctrine.

It is admissible to state, that any consistent mathematical physics of a specific, n-fold physical-spacetime manifold, can be read as if it were a formal, deductive theorem-lattice. In this interpretation, it appears that every theorem of that lattice has the qualifying at-

13. Formally, Euler's assertion was a defense of the purely arbitrary assumption of the naive Euclidean imagination, that linear extension is perfectly continuous without limit. Since Euler's supposed proof of that assertion depends absolutely upon the assertion of that axiom which it purports to prove, Euler's famous tautology proves nothing at all. Euler's folly on this point is the hereditary origin, via Lagrange and Laplace, of Cauchy's bowdlerization of Gottfried Leibniz's version of a calculus.

14. That attack upon Planck, first from within the German-speaking scientific community of the World War I interval, was continued in the savagery of Niels Bohr and other accomplices of Bertrand Russell, during the period of the famous 1920s Solvay Conference sessions.

15. This is literally an ancient issue. This topological challenge is the same ontological paradox, of the "One" and "Many," posed by Plato's *Parmenides.*

tribute of being a proposition which has been shown to be not-inconsistent with whatever set of axioms and postulates underlie that lattice in its entirety.[16] Such a set of axioms and postulates is identified by both Plato and Riemann as an *hypothesis*, in contrast to the illiterate's misuse of the same term in Newton's famous "*et hypotheses non fingo.*"[17]

The literate usage of "hypothesis," is mandatory in reading even the title of Riemann's June 1854 dissertation, even before proceeding to the body of the text. The key to a literate reading of Riemann's dissertation, is that a topological transformation typified by the transition from a mathematically *n*-fold physical-space-time manifold, to a manifold of (*n*+1) dimensions, is a transformation in the set of axioms and postulates underlying mathematical physics.

Consequently, the history of those discoveries of physical principle which, like Eratosthenes' discovery of an estimated curvature of the Earth, are validated by the relevant measurement, presents us with a succession of topological changes within mathematical physics, a series of changes which has the form of the "One"/"Many" paradox of Plato's **Parmenides.** In this instance, the "Many" are represented by a series of hypotheses; the challenge is to discover a higher principle, an *higher hypothesis*, a "One," which defines a generative principle by means of which the series of hypotheses, the "Many," is ordered "transfinitely." If Riemann's dissertation is read in any different sense than this platonic one, the resulting commentary upon the text is a scientifically illiterate one, no matter what the putative classroom authority of the commentator.

Riemann adopts a view of mathematical physics based upon the succession of advances in those discoveries of physical principle which have been validated crucially by relevant measurement, such as Eratosthenes' estimate for curvature of the Earth typifies that principle of measurement. Riemann's view of this topological transformation underlying mathematical physics' progress, thus defines progress in mathematical physics in terms of a sequence of absolute mathematical discontinuities within a formalist reading of mathematical physics itself. It defines Newton, Euler, and Cauchy, for example, as victims of their own scientific illiteracy, victims of an ontological paradox, of the "One"/"Many" form, which they could neither solve, nor comprehend—and, apparently, did not wish to comprehend.

In each case, one formal theorem-lattice is distinguished from another by any change in the axiomatic content, from that of the hypothesis underlying one, to that of the hypothesis underlying the other; every theorem of the second lattice is formally inconsistent with any theorem of the first. The difference between the two hypotheses, is a true, and relatively absolute mathematical discontinuity. Such a "discontinuity" has the same significance in mathematical physics as the proper understanding of the term "metaphor" in Classical forms of poetry or drama. What "discontinuity" signifies respecting the formalities of a consistent mathematical physics, is precisely what "metaphor" signifies for a Classical poem or drama.[18] The understanding of this relationship between metaphor and mathematical discontinuity, is the key to the first of the posthumously published documents, "On Psychology & Metaphysics," presented in the following pages.

In physics, a mathematical discontinuity appears as a mere mark. The magnitude of this mark is of *transinfinitesimal* smallness, so small that no calculable arithmetic magnitude can measure it, yet it exists, nonetheless, as a phenomenon: apparently as a mark of separation of all magnitudes which are less, from all magnitudes which are greater.[19] This mark signifies the functional presence, outside the realm of mathematical formalities, of the mathematical-physical form of what we recognize in Classical poetry as a metaphor.

16. E.g.: What Euler defends, by means of a rather silly tautology, in his 1761 attack upon Leibniz, is the naive, Euclidean, axiomatic assumption of the perfect persistence of linearization indefinitely, into the very large and very small.

17. **Riemann Werke**, p. 525: "*Das Wort Hypothese hat jetzt eine etwas andere Bedeutung als bei Newton. Man pflegt jetzt unter Hypothese Alles zu Erscheinungen Hinzugedachte zu verstehen.*"

18. The relevant problem is that, many miseducated readers with advanced degrees in arts have the same difficulty in coping with the term "metaphor," which radical positivists experience with the term "mathematical discontinuity." Beginning the early Seventeenth Century, the empiricists, such as Thomas Hobbes, launched a vile, energetic, and persisting campaign to eradicate the use of metaphor and the subjunctive mood from English-language usage. The recent emergence of that radical-existentialist decadence known as the "deconstructionism" of Professor Jacques Derrida, et al., is the outgrowth of a centuries-long campaign by the empiricists and logical positivists, and related linguistics specialists, to locate the origin of written language, even Classical poetry, in "text" as such, rather than the irony-rich domain of speech.

19. In the extremely small, discontinuities are compared in respect to their mathematical cardinality, not as arithmetic values. Hence, with deference to Georg Cantor, this distinction is designated here by the usage of "transinfinitesimally small."

Riemann's 'Geistesmassen'

The fact that all true metaphors are singularities, is the key to an accurate understanding of Riemann's use of *Geistesmassen*, translated here as "thought masses," in the first of the posthumously published papers, "On Psychology and Metaphysics." As an illustration of the principle involved, consider the case of metaphor in either a Classical form of strophic poem, or a song-setting of such a poem by a Mozart,[20] Beethoven, Schubert, Schumann, or Brahms.[21] This case, of the Classical strophic poem, and its musical setting according to principles of motivic thorough-composition, is key for understanding the mental processes by means of which a validatable discovery of new scientific principle is generated.[22] This is also an example of the conception posed by Plato's treatment of the "One/Many" ontological paradox in his **Parmenides** and other late dialogues.[23]

In the successful Classical poem, efficiently illustrated as to form by Goethe's simple **Mailied**,[24] the strophes represent a succession of metaphors, which march, one after the other, toward a conclusion. The metaphorical attribution of each of those strophes is generated by ironies, to such effect that no proper attribution of either a confining literal or a symbolic meaning for that strophe is to be permitted. The concluding metaphor, especially its final couplet, changes radically the metaphorical attribution—e.g., the "meaning"—of the poem as a whole. It is that concluding, subsuming metaphor, which identifies the idea of the poem taken in its entirety.

The literate reading of such a poem, or its Classical song-setting, demands a repeated review of the completed poem, until the point is reached that two conditions are satisfied: first, that the idea of the completed poem as a whole is clear; second, that the relationship of each step of progress within the poem, to the reaching of the conclusion, is clear.[25] The satisfaction of that requirement establishes the idea of the poem as a whole, in the mind, as the product of a tension between two, literally platonic qualities of idea. The first, is the idea of the completed poem in its entirety; this idea remains unchanged, from prior to the re-reading of the first line, to the momentary silence following the reading of the last line. The second idea, is the successive metamorphoses which the idea of the poem undergoes, in proceeding from the beginning to the end. In Plato, that latter quality of idea is identified as the *Becoming*. It is the tension between the fixed conception, the idea of the completed poem as a whole, and the metamorphical character of the process of Becoming, by which the perfected idea is reached, which is the "energy" of the poem.

The same requirement applies to the performance of any Classical musical composition. In the simplest case of such a musical performance, it is the performer's memory of reaching the perfected (completed) composition, which creates the tension of reenacting the performance of the metamorphosis, the tension between the perfected idea of the composition, and the moment of development in mid-performance.

The singularity in question is generated by the difference in direction of time-sense—backwards versus forwards—of the two, interacting ideas respecting the poem or musical composition in mid-performance.

The same principle characterizes Eratosthenes' estimate of the curvature of the Earth's surface: the principle of development uncovered, by re-experiencing the mutually contradictory individual readings of the midday sundials, to locate a generating principle of change which is consistent with the final result. For Eratosthenes, the key to the generating principle becomes the relationship between the perimeter of a circle and a pencil of lines, from a momentarily fixed position of the point corresponding to the Sun, to the Earth. Thus, Eratosthenes gave a reasonable estimation of the Earth's curvature, approximately twenty-two centuries before

20. After Mozart's first song composed in the new mode of motivic thorough-composition, his setting of Johann Goethe's "Das Veilchen" ("The Violet"). See **A Manual on the Rudiments of Tuning and Registration**, John Sigerson and Kathy Wolfe, eds. (Washington: Schiller Institute, 1992), Chapter 11, pp. 199-228.

21. Op. cit., pp. 220-221. Note the reference to Gustav Jenner, **Johannes Brahms als Mensch, Lehrer und Künstler: Studien und Erlebnisse** (Marburg an der Lahn: N.G. Elwert'sche Verlagsbuchhandlung, 1930). Jenner's account of Brahms' instruction to him on composing a song for a strophic poem, is directly relevant to the point being developed at this point in the text, above.

22. See Lyndon H. LaRouche, Jr., "Musical Memory and Thorough-Composition," *Executive Intelligence Review*, Sept. 1, 1995, pp. 50-63.

23. Plato's *Parmenides* is to be considered as a kind of prefatory piece for all of his later dialogues. In it, he poses the challenge, the ontological paradox, which is the subject addressed in its various aspects by all of the other late dialogues.

24. LaRouche, "Musical Memory and Thorough-Composition," p. 55. See note 22.

25. See Jenner's account of his instructions from Brahms, on memorizing a poem with sufficient thoroughness to satisfy those requirements, before undertaking to provide a song-setting for it. See note 21.

any person saw that curvature.

These examples, from poetry, music, and the work of Plato's Academy of Athens, are each and all examples of *platonic ideas*, the quality of ideas to which Riemann assigns the term *Geistesmassen*. In physical science generally, such ideas have initially the apparent character of ideas arising from vicious inconsistencies within observations made by aid of sense-perception, inconsistencies which mock both naive sense-certainty and generally accepted scientific opinion. Relatively often, that mockery occurs in the most cruelly devastating way. Those ideas which purport to identify the generating principle responsible for this paradox, and which are validated by relevant modes of measurement, represent valid discoveries of physical principle. Those qualities of proven principle are classically identified as *platonic ideas*. Each and all of the validated ideas of "dimensionality" in an *n*-fold physical-space-time manifold, have this quality of platonic idea.

Thus, all such ideas have the form of paradoxical singularities relative to the pre-existing mathematical domain of reference. The character of these ideas as singularities arises from the way in which their existence is generated *subjectively*: by the same kind of processes underlying the reading and composition of a valid Classical strophic poem. The quality of "singularity," and the associated form of mathematical discontinuity, arises from the opposing senses of time associated with the interplay of perfected ideas with the process of their development.[26]

These metaphors can never be deduced from the mathematics, or other form of language employed. Within the language itself, they appear merely in the reflected form of singularities, such as either mathematical discontinuities or other paradoxical adumbrations reflected into the language-medium. The ontological existence of the singularity lies outside the form of generation of the relevant mark within the domain of the language itself.

Thus, every theorem which claims to deny the existence of discontinuities within mathematics, such as Euler's, is based upon *the tautological fallacy of composition, of using constructions premised axiomatically on linearization, to prove the utterly irrelevant point, that any construction of this type is incapable of acknowledging any mathematical existence which is not linear!*

26. The proper notions of topology are derived from this consideration.

The relevant formal mathematical discontinuity, or literary paradox, is merely the mark which the metaphor imposes, as its footprint, upon the formally defined medium of language. The actual metaphor, which the adumbrated mark, or paradox reflects, exists only outside the medium. It lies within three locations. It lies, first, in the substance of the process which the language is attempting to describe. It also lies, secondly, in the mental processes of the scientist, or the artist. It exists, thirdly, within the sovreign mental processes of those members of the audience who have responded Socratically to the mark of the singularity, by generating in their own mind a replication of the idea which has imposed its mark upon the medium of communication.

In mathematical physics, the validation of the ideas corresponding to such marks occurs commonly through measurements which demonstrate, that those ideas correspond efficiently to an effect which is not in correspondence with the old ideas which the new ideas profess to supersede.

There is a most notable illustration of this point in the case of Riemann's paper, published in 1860, "On the Propagation of Plane Air Waves of Finite Amplitude."[27] The fact that acceleration toward speeds above the speed of sound generates a singularity, was recognized by Riemann as showing the existence of the transsonic phenomena studied by such followers as Ludwig Prandtl and Adolf Busemann. It was this principle of Riemann's which resulted, through the mediation of a German aerospace specialist, in the first successful powered, post-World War II, supersonic flight by a U.S. aircraft. This was in contrast to the failed contrary opinion expressed by such frequent adversaries of Riemann's work as Hermann Helmholtz, Lord Rayleigh, and Theodor von Karman.[28]

27. "*Über die Fortpflanzung ebener Luftwellen von endlicher Schwingungsweite*," **Riemann Werke**, pp. 156-175. This was published in an English translation by Uwe Henke and Steven Bardwell, in the Fusion Energy Foundation's *International Journal of Fusion Energy*, Vol. 2, No. 3, 1980, pp. 1-23.

28. There is a relevant story behind the Fusion Energy Foundation's publication of that translation. During the middle to late 1970s, the Fusion Energy Foundation (FEF) gained an international reputation for its important work in promoting inertial confinement fusion. As a consequence of this, in 1978, two representatives of the FEF, Mr. Charles B. Stevens, Jr., and Dr. Steven Bardwell, were invited to the Soviet Union to participate in an international scientific conference on inertial confinement. Prior to their departure, these two FEF representatives met with LaRouche and others, at a Bronx location, to obtain LaRouche's list of requirements for that Moscow visit. LaRouche requested that they ask Soviet scientists for unclassified documents pertaining to the

In the relatively more obvious type of case, such as the cited Eratosthenes case, the empirical validation of such a singularity is accomplished by measurements which lie within the domain of arithmetic magnitudes. However, this is not the only primary form of empirical proof of a platonic idea. As Riemann's referenced paper on shock-waves illustrates the point, in some cases, it is the existence of a non-arithmetic singularity, which has precise cardinality, but not arithmetic magnitude, which presents us the mathematical form of the required proof. Riemann's success in forecasting a class of phenomena not necessarily limited to this cited case, not only powered transsonic/supersonic flight, but isentropic compression in thermonuclear ignition, is an example of this.

Leibniz's Universal Characteristic

Respecting the ontological implications of metaphor itself, within these posthumously published pieces, Riemann picks up on a theme addressed earlier by Leibniz, and later revived by the present writer. We must consider the fact, that those efficient platonic ideas recognizable as validated discoveries of principle, are generated as discoveries within those sovreign mental processes of the individual which are impenetrable by symbolic communications-media, such as a formal mathematics. Yet, despite the ethereal quality one might be tempted to attribute wrongly to such mental processes, the result of such ideas is an increase of the human species' physical power to command nature in general.

In this respect, these papers of Riemann turn our attention back to Leibniz's notion of a *Universal Characteristic*, which subsumes, commonly, non-living, living, and cognitive processes within our universe. This is the topical area addressed in the first two of the posthumously published papers: "I. On Psychology and Metaphysics," and "II. Epistemological issues." After the writing of these papers, Riemann's published work does not refer explicitly again to such epistemological underpinnings of science. From 1854 on, his published work limits itself essentially to mathematical physics, with some impingement upon biophysics,[29] although he clearly did not abandon that personal standpoint in his thinking about mathematical-physics matters. Therein lies some of the special importance of the posthumously published papers for identifying the deeper implications of Riemann's work as a whole.

My own discoveries in physical-economy were rooted in my youthful profession as a follower of Leibniz, and in my developing a rigorous defense of Leibniz against Immanuel Kant's attacks upon him, the latter a matter which bears directly upon the issue of Leibniz's notion of a Universal Characteristic. Furthermore, my discoveries were provoked by both the positivist excesses of Norbert Wiener's "information theory" and the similar incompetence of the work in systems analysis by one of Wiener's followers, John von Neumann; these positivist concoctions I had treated as parodies of Kant's attack on Leibniz. For this reason, my rereading of Riemann brought to that reading the same emphasis upon Leibniz's Universal Characteristic which we encounter in the first two items among Riemann's posthumously published pieces.

The kernel of Wiener's hoax in "information theory," was to adopt and misuse a term, "negative entropy," which had been used earlier chiefly to identify the qualitative distinction between living and non-living processes as they present themselves on the scale of macrophysics.[30]

In successful modern physical economies, my field of study, the biological appearance of "negative entropy" is echoed by the requirement that the ratio of relative "free energy" to "energy of the system" must not decrease, despite the accompanying requirement of rising per-capita and per-square-kilometer values of capital-intensity and power-intensity. This desired

use of Riemann's work on isentropic compression as a basis for the original development of thermonuclear ignition. Such unclassified documentation was obtained, identifying this Riemann *Fortpflanzung* paper in that connection. It was at a subsequent, "report back" meeting that same year, that LaRouche underlined the application of the same paper to physical-economic modelling, and presented the set of inequalities used to create the highly successful 1980-1983 U.S. Quarterly Economic Forecast of the *Executive Intelligence Review* (**EIR**) newsweekly.

29. E.g., the brilliantly confirmed analysis provided within his *Mechanik des Ohres* (Mechanics of the Ear): **Riemann Werke**, pp. 338-350.

30. As noted, repeatedly, in other locations, this reporter has found it desirable to apportion all physical science among four functionally distinguished domains of inquiry. Two areas, astrophysics and microphysics, are domains in which the scale of phenomena is either too large, or too small, to be addressed directly by the senses. In a third area, biophysics, we deal with the principled distinction between processes, such as organic compounds, which, in one instant are functioning as part of a living process, and, in another instant, not. This also defies simple sense-perception. Those three domains, leave, as residue, the domain of macrophysics, in which sense-perception plays a larger immediate role.

result is realized, typically, by the fostering of increase of the (physical) productive powers of labor through investment in scientific and technological progress.

Consider the following summary of the relevant argument elaborated in other locations.[31]

Physical economy identifies the primary phenomena of economic processes in terms of market-baskets of both necessary physical consumption and certain crucial classes of services, limited essentially (in modern society) to education, health care, and science and technology as such. These market-baskets are defined per capita (of labor-force), per household, and per square kilometer of relevant land-area employed. The market-baskets are defined for personal consumption, for the processes of production, and for those improvements in land-area used which we class under "basic economic infrastructure." Physical economy recognizes a required functional relationship between the level of these market-baskets and the productive powers of labor, as measured in terms of both production and consumption of the content of these market-baskets.[32]

That yields an implied differential expression: What level of input (consumption) is required to maintain a certain rate of output of necessary products for consumption? Without yet knowing the exact answer to that question at any given point, the idea of the question is clear. This idea is expressed conveniently as the notion of *potential relative population-density*.[33]

The levels of combined market-basket consumption which are required to maintain not less than some constant rate of potential relative population-density, are compared to the notion of "energy of the system." Output of market-basket content in excess of those required levels, is compared to "free energy." The "free energy" is considered "not wasted," on the condition that it is consumed in market-basket forms, for both expanding the scale of the economy, and increasing the potential relative population-density. It the latter case, the capital-intensity ("energy of the system" per capita, per household, and per square kilometer) must increase, and the power-density must also increase. The requirement is, that the ratio of apparent "free energy" to "energy of the system" must not decrease, despite a rising relative value of "energy of the system" per capita, per household, and per square kilometer.

The increase of potential relative population-density, under the condition that those constraints are satisfied, is treated as the economic-process analog for what is expressed as "negative-entropic" evolutionary self-development of the biosphere in biology and in the terms of reference supplied by the Academician V.I. Vernadsky's notion of biogeochemistry. To avoid confusion with the "information theory's" popularized misuse of the term "negative entropy," the term "not-entropy" is employed instead.

In the field of what Academician V.I. Vernadsky defined as biogeochemistry, this requires the evolution of the biosphere, to bring the entire system to a higher state of organization; Vernadsky's argument typifies the line of thought which is otherwise encountered in various locations, including Leibniz's notion of a Universal Characteristic, and also the referenced portions of Riemann's posthumously published papers.

Wiener made a mess of everything, with the popularization of his wretched insistence that "negative entropy," for which he employed the neologism "negentropy," was no more than a reversal of the statistical entropy described by Ludwig Boltzmann's H-theorem. Contrary to Wiener's mechanistic schemes, if we account for mankind and mankind's activity as part of the planetary system, man's increased power over nature, typified by the increase of mankind's potential relative population-density,[34] is actually an increase of the relative "negative entropy," or, "not-entropy," of the planetary system as a whole. In other words, mankind's development supplies an evolutionary upward impulse to the totality of the system with which mankind interacts.

In this view of the matter, human cognition has developed within the domain of living processes, but those ecological characteristics of the human species

31. E.g., Lyndon H. LaRouche, Jr., "Why Most Nobel Prize Economists Are Quacks," and "Non-Newtonian Mathematics for Economists." See note 6.

32. E.g., the case for household consumption was indicated by Gottfried Leibniz in **Society and Economy** (1671), which appears in English translation in **Executive Intelligence Review,** Jan. 4, 1991, pp. 12-13.

33. On "relative population-density," see Lyndon H. LaRouche, Jr., **So, You Wish to Learn All About Economics?** (New York: New Benjamin Franklin House, 1984). This introductory textbook has been published in various languages, including Russian, Ukrainian, and, most recently, Armenian.

34. per capita of labor-force, per household, and per square kilometer of relevant land-area employed.

which are entirely due to cognition, place mankind absolutely apart from and above all other living species. Thus, our universe subsumes the interaction among three distinguishable types of processes: non-living, living, and cognitive. The commonly subsuming principle governing such a universe, is Leibniz's notion of a Universal Characteristic.

For today's conventional classroom opinion, what we have just stated poses the question: "Is it not necessarily the case, that if the 'not-entropy' of society increases, that this must occur at the price of increasing the entropy of the universe with which society is interacting?" In other words, is the relationship of society to the remainder of the universe not what von Neumann's devotees term "a zero-sum game"? The crux of the issue, is that the idea of "universal entropy" is not a product of scientific discovery, but of the reckless application of an axiomatically linear, mechanistic world-view, upon the interpretation of the evidence of kinematic models of gases; on this account, there is an amusing ambiguity in the ironical meaning Norbert Wiener's work supplies to the term "gas theory."

The absurdity of the popular version of doctrines of "universal law of entropy," is suggested by the fact, that every rational effort to describe the universe in the large, is an evolutionary model, in which development is vectored as progress to relatively higher states of organization. In mathematical terms, this progress to higher states of organization is indicated by the emergence of physical systems whose characteristics can not be identified without resort to the mathematics of successively higher cardinalities. The attempt to explain the efficient directedness of such universalizing processes of emergence of higher cardinalities, renders absurd every attempt to explain the existence of matter itself in terms of a mechanistic dogma of "building blocks." The evidence is, that recognizably higher physical states of cardinality, are accomplished by transformations of the entire system, not by accretions of objects of a mechanistically fixed domain.

The counterposing of the developmental (e.g., not-entropic) and Kant-like mechanistic views is noted by Riemann, in the first of the referenced papers. Crucial is the demonstration, that, as in the case of Euler's absurd 1761 attack on Leibniz's **Monadology**, the presumption of that Kant-like, mechanistic view, from which Richard Clausius, Lord Kelvin, and Hermann Grassmann concocted their chimerical "Second Law of Thermodynamics,"[35] is "axiomatic linearization in the small." Create a mathematics, in which all is subsumed under the axiomatic assumption, that everything in the universe is consistent with the Euclidean blind faith in the universality of perfectly continuous linear extension, even into the extremely great and the extremely small. The true believer then regards any formulation which is inconsistent with such a mathematical "proof," as "disproven," and everything which must be assumed to preserve consistency within the theorem-lattice of such a mathematics, is considered as "proven" by all of the awesomely credulous professorial, head-nodding dupes attending the relevant conference.[36]

Once we recognize, that such a mathematics constitutes no proof at all respecting the issues immediately at hand, the most generous consideration which the advocates of the "Second Law" might require of rational people, is the famous Scots' verdict, "not proven." No axiom of a mathematics is proven by the employment of the formal mathematical theorem-lattice whose existence depends upon that included assumption.

Those qualifying observations stated, situate the matter at hand. Now, turn directly to the subject of Leibniz's Universal Characteristic.

The paradigmatic form of all increase in mankind's

35. It was Kelvin who proposed to Clausius this radically mechanistic interpretation of Sadi Carnot's work. In this case, as in all of his attacks upon Bernhard Riemann, Clausius relied upon Hermann Grassmann for the mathematical side of his endeavors. See **Riemann Werke**, note on page 293. The crucial role which the axiomatic presumption of linearization in the small played in Grassmann's work, including all of his work on the "Second Law" and attacks upon Riemann, is reflected in his famous 1844 work founding a relevant branch of modern vector analysis, the so-called *Ausdehnungslehre.*

36. During 1978, former FEF Director Morris Levitt dug out a document authored by J. Clerk Maxwell which caused FEF much amusement at that time. In this document, Maxwell responded to the question: Why had Maxwell failed to give credit to such predecessors as Wilhelm Weber and Riemann (and also, most crucially, the founder of electrodynamics, Ampère) for many of the discoveries which Maxwell tacitly presented as either the work of Michael Faraday, or his own? To this, Maxwell replied, that "we," referring to the circles including Kelvin, et al., had chosen to disregard any work which relied upon geometries "different than our own." The same point is made, in similar terms, in Maxwell's principal work. The implication of Lord Rayleigh's denunciation of Riemann's *Fortpflanzung* paper, is the same: the root of the mechanistic world-view, which the empiricist world-outlook of modern Britain acquired from its ancient master, Paolo Sarpi, is always the presumption of the universality of percussive causality within a universe which is axiomatically linearized in the very small.

potential relative population-density, from the several millions potential of a man-like higher ape, to the billions of today, is changes in social-productive behavior typified by general application of the fruits of scientific and technological progress.[37]

Each of the transmitted discoveries is known by means of the replication of that original act of discovery within the mind of the hearer. On the condition that education of the young proceeds according to that latter principle, present-day knowledge is the accumulation of all of those singularities which valid past discoveries have conveyed to the use of the present generations: just as students today would be scientific illiterates, until they re-experience the original discoveries by the members of Plato's Academy at Athens in this way, from Plato, Eudoxus, and Theaetetus, through Eratosthenes. Without a Classical education of the young, in the great Classical works of poetry, tragedy, music, and natural science, going back to the foundations of modern civilization over 2,500 years ago, there can not be a truly civilized or even rational society, a cruel fact we see enacted so brutishly on our streets and in our government and universities today.

Each valid such discovery invokes the principle we have associated here with the topological symbol $(n+1)/n$. Each discovery is a singularity of that type. Progress in knowledge is an accumulation of such singularities. As Riemann emphasizes, within the texts provided below, that accumulation of knowledge is interactive, every new concept interacting with every other accumulated within the same mind. Thus, with every thought, this increase of singularities is reflected efficiently: in mathematical terms, the density of discontinuities for any arbitrarily selected interval of human action, is increased. It is this increase of "density of discontinuities" which typifies the form of "not-entropic" and the form of the action which generates "not-entropy" in, for example, the form of increase of society's potential relative population-density.

The crucial fact is, that this increase of knowledge, as defined in this way, is consistently efficient. *The universe obeys the human creative-mental powers' command!* Thus, as **Genesis 1** prescribes, mankind exerts dominion over nature. Conversely, the universe is manifestly so constituted, that it is prone to submit to the authority of that power of creative reason which is a potentiality peculiar to the individual human personality.

By accumulating a reliving of the original valid acts of discovery of principle, which constitute the accumulation of human knowledge to the present date, we are enabled to recognize the distinguishing features of that form of act of creative reason, by means of which valid discoveries have been commonly achieved. That experience becomes known to us, as to Johannes Kepler, as *Reason*, or, as for Gottfried Leibniz, as *necessary and sufficient reason*. Once we recognize, that mankind's cumulative development of knowledge represents the power of the human will to command the universe according to the law embedded in that universe, we have shown ourselves that *reason* as we define it *subjectively* in this way, is also an efficient approximation of Reason as it exists, ostensibly *objectively*, as an efficient principle pervading the universe as a whole.

What we recognize in the form of "not-entropy," as in the increase of society's potential relative population-density, is the characteristic of Reason, both as it exists efficiently, "objectively" within the universe at large, and as we are able to adduce the principles of reason, "subjectively," through the efficiency of valid discoveries of principle in the domains of science and art.

Once that is acknowledged, then it is clear to us, that the universe is not linearized in the extremely small, or extremely large. It is "not-entropic," in the extremely small and extremely large, alike. To see this more clearly, it was sufficient, to shift the emphasis in reading Riemann's contributions to mathematical physics, away from physics narrowly conceived, back to the vantage-point of Leibniz, the vantage-point of physical economy, the vantage-point of the efficient relationship between valid human individual reason, and man's increased power over the universe. Thus, we may say, that not-entropy, as reflected in type by Riemann's topological expression $(n+1)/n$, corresponds to what Leibniz named a Universal Characteristic.

37. This progress in the human condition is not due only to scientific and technological progress. The metaphors which arise from Classical forms of poetry, tragedy, and music have as crucial a role in increasing man's power to exist as what we term conventionally "natural science." Nonetheless, as we have already indicated, valid fundamental scientific discoveries merely typify the more general case for all forms of expression of the creative-mental powers of persons as metaphor: as the great English poet Percy Shelley expressed the point, within his "A Defence of Poetry": the "power of communicating and receiving intense and impassioned conceptions respecting man and nature." What is stated above, here, should be read with the understanding that the case for scientific ideas *typifies* the case for metaphor in general.